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**New York State Testing Program
Regents Examination in
Algebra I (Common Core)**

Selected Questions with Annotations

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DRAFT



THE STATE EDUCATION DEPARTMENT /
THE UNIVERSITY OF THE STATE OF NEW YORK / ALBANY, NY 12234

New York State Testing Program Regents Examination in Algebra I (Common Core)

Selected Questions with Annotations

With the adoption of the New York P-12 Common Core Learning Standards (CCLS) in ELA/Literacy and Mathematics, the Board of Regents signaled a shift in both instruction and assessment. In Spring 2014, New York State administered the first set of Regents Exams designed to assess student performance in accordance with the instructional shifts and the rigor demanded by the Common Core State Standards (CCSS). To aid in the transition to new tests, New York State released a number of resources during the 2013-2014 year, including sample questions, test blueprints and specifications, and criteria for writing test questions. These resources can be found at <http://www.engageny.org/resource/regents-exams>.

New York State administered the first English/Language Arts and Algebra I Common Core Regents Exams in June 2014 and is now annotating a portion of the questions from those tests available for review and use. These annotated questions will help students, families, educators, and the public better understand how tests have changed to assess the instructional shifts demanded by the Common Core and to assess the rigor required to ensure that all students are on track to college and career readiness.

Annotated Questions Are Teaching Tools

The annotated questions are intended to help students, families, educators, and the public understand how the Common Core is different. The annotated questions will demonstrate the way the Common Core should drive instruction and how tests have changed to better assess student performance in accordance with the instructional shifts demanded by the Common Core. They are also intended to help educators identify how the rigor of the Regents Examinations can inform classroom instruction and local assessment. The annotations will indicate common student misunderstandings related to content standards; educators should use these to help inform unit and lesson planning. In some cases, the annotations may offer insight into particular instructional elements (conceptual thinking, mathematical modeling) that align to the Common Core that may be used in curricular design. It should not be assumed, however, that particular standards will be measured with identical items in future assessments.

The annotated questions include both multiple-choice and constructed-response questions. With each multiple-choice question annotated, a commentary will be available to demonstrate why the question measures the intended standards; why the correct answer is correct; and why each wrong answer is plausible but incorrect. The rationales describe why the wrong answer choices are plausible but incorrect and are based in common misconceptions or errors in computation. While these rationales speak to a possible and likely reason for selection of the incorrect option by the student, these rationales do not contain definitive statements as to why the student chose the incorrect option or what we can infer about knowledge and skills of the student based on their selection of an incorrect response. These multiple-choice questions are designed to assess student proficiency, not to diagnose specific misconceptions/errors with each and every incorrect option.

For each constructed-response question, there will be a commentary describing how the question measures the intended standards, and sample student responses representing possible student errors or misconceptions at each possible score point.

The annotated questions do not represent the full spectrum of standards assessed on the State test, nor do they represent the full spectrum of how the Common Core should be taught and assessed in the classroom. Specific criteria for writing test questions as well as test information are available at <http://www.engageny.org/resource/regents-exams>.

Understanding Math Annotated Questions

All questions on the Regents Exam in Algebra I (Common Core) are designed to measure the Common Core Learning Standards identified by the PARCC Model Content Framework for Algebra I. More information about the relationship between the New York State Testing Program and PARCC can be found here: <http://www.p12.nysed.gov/assessment/math/ccmath/parccmcf.pdf>.

Multiple Choice

Multiple-choice questions are designed to assess CCLS for Mathematics. Mathematics multiple-choice questions will mainly be used to assess students' procedural skills and conceptual knowledge. Multiple-choice questions incorporate both Standards for Mathematical Content and Standards for Mathematical Practice and some real-world applications. Many multiple-choice questions require students to complete multiple steps. Likewise, many of these questions are linked to more than one standard, drawing on simultaneous application of multiple skills and concepts. Within answer choices, distractors will all be based on plausible missteps.

Constructed Response

Constructed-response questions will require students to show a deep understanding of mathematical procedures, concepts, and applications. The Regents Examination in Algebra I (Common Core) contains 2-, 4-, and 6-credit constructed-response questions.

2-credit constructed-response questions require students to complete a task and show their work. Like multiple-choice questions, 2-credit constructed-response questions will often involve multiple steps,

the application of multiple mathematics skills, and real-world applications. These questions may ask students to explain or justify their solutions and/or show their process of problem solving.

4-credit and 6-credit constructed-response questions require students to show their work in completing more extensive problems that may involve multiple tasks. Students will be asked to make sense of mathematical and real-world problems in order to demonstrate procedural and conceptual understanding. For 6-credit constructed-response questions, students will analyze, interpret, and/or create mathematical models of real-world situations.

#2

2 Officials in a town use a function, C , to analyze traffic patterns. $C(n)$ represents the rate of traffic through an intersection where n is the number of observed vehicles in a specified time interval. What would be the most appropriate domain for the function?

- (1) $\{\dots -2, -1, 0, 1, 2, 3, \dots\}$ (3) $\left\{0, \frac{1}{2}, 1, 1\frac{1}{2}, 2, 2\frac{1}{2}\right\}$
(2) $\{-2, -1, 0, 1, 2, 3\}$ (4) $\{0, 1, 2, 3, \dots\}$

Measured CCLS Cluster: F-IF.B

Key: (4)

Commentary: This question measures the knowledge and skills described by the standards within F-IF.B because it requires the student to relate the domain of a function to the quantitative relationship that it describes. The function in this question, C , models the relationship between the number of observed vehicles in a specified time interval and the rate of traffic through an intersection. The student must identify the domain that most appropriately represents the number of observed vehicles, n . This item does not require students to perform a specific calculation or procedure; rather, the student must understand the relationship between a model and the real world situation it describes, employing MP.2 (Reason abstractly and quantitatively) and MP.4 (Model with mathematics).

Answer Choice (1): $\{\dots -2, -1, 0, 1, 2, 3, \dots\}$ This response is incorrect and represents an inappropriate domain for the function. Since it is stated that n represents the number of observed vehicles in a specified time interval, negative integers are not appropriate to include in the domain. The student may have understood that the domain should include whole number values, but did not understand that negative integers would be inappropriate, based on the quantitative relationship being described. A student selecting this choice may have a limited understanding of how the domain of a function relates to the quantitative relationship it describes.

Answer Choice (2): $\{-2, -1, 0, 1, 2, 3\}$ This response is incorrect and represents an inappropriate domain for the function. Since it is stated that n represents the number of observed vehicles in a specified time interval, negative integers are not appropriate to include in the domain. Additionally, it is not reasonable to assume that the number of vehicles recorded would be limited to 3. The student may have confused the concepts of domain and range or lacked understanding of how the function in this question relates to the situation described. A student selecting this choice may have a limited understanding of how the domain of a function relates to the quantitative relationship it describes.

Answer Choice (3): $\{0, \frac{1}{2}, 1, 1\frac{1}{2}, 2, 2\frac{1}{2}\}$ This response is incorrect and represents an inappropriate domain for the function. Since it is stated that n represents the number of observed vehicles in a specified time interval, non-whole numbers are not appropriate to include in the domain. The student may have confused the concepts of domain and range or lacked understanding of how the function in this question relates to the situation being described. A student selecting this choice may have a limited understanding of how the domain of a function relates to the quantitative relationship it describes.

Answer Choice (4): $\{0, 1, 2, 3, \dots\}$ This response is correct and represents the most appropriate domain for the function. Since the domain of the function is based on the number of cars observed, it is most appropriate that the domain consist of nonnegative integers. A student selecting this choice understands how the domain of a function relates to the quantitative relationship it describes.

Rationale: Choices (1), (2), and (3) are plausible but incorrect. They represent common student errors made when determining the domain of a function. Choosing the correct solution requires students to know how to relate the domain of a function to the quantitative relationship it describes. Compare with questions #9 and #18, which also assess F-IF.B.

#6

6 The table below shows the average yearly balance in a savings account where interest is compounded annually. No money is deposited or withdrawn after the initial amount is deposited.

Year	Balance, in Dollars
0	380.00
10	562.49
20	832.63
30	1232.49
40	1824.39
50	2700.54

Which type of function best models the given data?

- (1) linear function with a negative rate of change
- (2) linear function with a positive rate of change
- (3) exponential decay function
- (4) exponential growth function

Measured CCLS Cluster: F-LE.A

Key: (4)

Commentary: This question measures the knowledge and skills described by the standards within F-LE.A because the student must recognize a situation in which a quantity grows by a constant percent rate per unit interval relative to another. Inherent in these standards is the distinction between linear functions, which grow by equal differences over equal intervals, and exponential functions, which grow by equal factors over equal intervals. In this situation, the table shows that over regular intervals of 10 years, the “Balance, in Dollars” increases by a constant percent rate or by an equal factor, which indicates exponential growth. This question requires students to employ MP.4, as they consider and evaluate models for a real-world situation

Answer Choice (1): Linear function with a negative rate of change. This response is incorrect and is a description of a function that would not accurately model the given data. Since the data do not show a constant rate of change and the balance is increasing over time, a linear model with a negative rate of change is not appropriate. A student who selects this response may have confused the concepts of linear and exponential functions. A student who selects this response may have a limited understanding of how to recognize a situation in which a quantity grows by a constant percent rate per unit interval relative to another.

Answer Choice (2): Linear function with a positive rate of change. This response is incorrect and is a description of a function that would not accurately model the given data. Since the data do not show a constant difference per interval, a linear model is not appropriate. A student who selects this response may have some understanding of functions and may recognize that the amount of money is growing positively, but may not understand that the money is not growing at a constant rate per unit interval or may have confused the concepts of exponential and linear functions. A student who selects this response may have a limited understanding of how to recognize a situation in which a quantity grows by a constant percent rate per unit interval relative to another.

Answer Choice (3): Exponential decay function. This response is incorrect and is a description of a function that would not accurately model the given data. While the data do indicate an exponential function, since the data are increasing, a decay function is not appropriate. A student who selects this response may be able to recognize that the data can be modeled by a nonlinear function, but cannot recognize that the function should be increasing.

Answer Choice (4): Exponential growth function. This response is correct and represents the description of a function that would accurately model the given data. The student correctly determined that the function represented is an exponential growth function as the balance increases at a constant percent rate per unit interval. A student who selects this response understands how to recognize a situation in which a quantity grows by a constant percent rate per unit interval relative to another.

Rationale: Choices (1), (2), and (3) are plausible but incorrect. They represent common student errors made when required to recognize a situation in which a quantity grows by a constant percent rate per unit interval relative to another. Choosing the correct solution requires students to know the difference between linear and exponential functions as well as the difference between a positive and negative rate of change. Compare with questions #15 and #24, which also assess F-LE.A.

#8

8 Which equation has the same solution as $x^2 - 6x - 12 = 0$?

(1) $(x + 3)^2 = 21$

(3) $(x + 3)^2 = 3$

(2) $(x - 3)^2 = 21$

(4) $(x - 3)^2 = 3$

Measured CCLS Cluster: A-REI.B

Key: (2)

Commentary: This question measures knowledge and skills described by the standards within A-REI.B because the student must identify a correct transformation of a given quadratic equation into an equation of the form $(x - p)^2 = q$, with the same solution. Rewriting a quadratic equation in this form can be accomplished by an algebraic method called “completing the square.”

Answer Choice (1): $(x + 3)^2 = 21$. This response is incorrect and is an equation that does not have the same solution as the given quadratic equation. The student may have made a computational error when transforming the given equation, possibly by incorrectly factoring the expression $x^2 - 6x + 9$ as $(x + 3)(x + 3)$ or may lack understanding of what it means for two equations to have the same solution. A student who selects this response may have a limited understanding of using completing the square to transform a given quadratic equation into an equation of the form $(x - p)^2 = q$.

$$x^2 - 6x - 12 = 0$$

$$x^2 - 6x + 9 = 12 + 9$$

$$(x + 3)^2 = 21$$

Answer Choice (2): $(x - 3)^2 = 21$. This response is correct and is an equation with the same solution to the given quadratic equation. A student who selects this response was able to use the method of completing the square in order to transform the given quadratic equation into the form $(x - p)^2 = q$.

$$x^2 - 6x - 12 = 0$$

$$x^2 - 6x + 9 = 12 + 9$$

$$(x - 3)^2 = 21$$

Answer Choice (3): $(x + 3)^2 = 3$. This response is incorrect and is an equation that does not have the same solution as the given quadratic equation. The student may have made a procedural error when transforming the given equation, possibly by subtracting 9 from both sides of the equation, or may lack understanding of what it means for two equations to have the same solution. A student who selects this response may have a limited understanding of how to transform a given quadratic equation into an equation of the form $(x - p)^2 = q$.

$$x^2 - 6x - 12 = 0$$

$$x^2 - 6x - 9 = 12 - 9$$

$$(x + 3)^2 = 3$$

Answer Choice (4): $(x - 3)^2 = 3$. This response is incorrect and is an equation that does not have the same solution as the given quadratic equation. The student may have made a computational error when transforming the given equation, possibly by incorrectly subtracting 9 from only the right side of the equation, or may lack understanding of what it means for two equations to have the same solution. A student who selects this response may have a limited understanding of how to transform a given quadratic equation into an equation of the form $(x - p)^2 = q$.

$$x^2 - 6x - 12 = 0$$

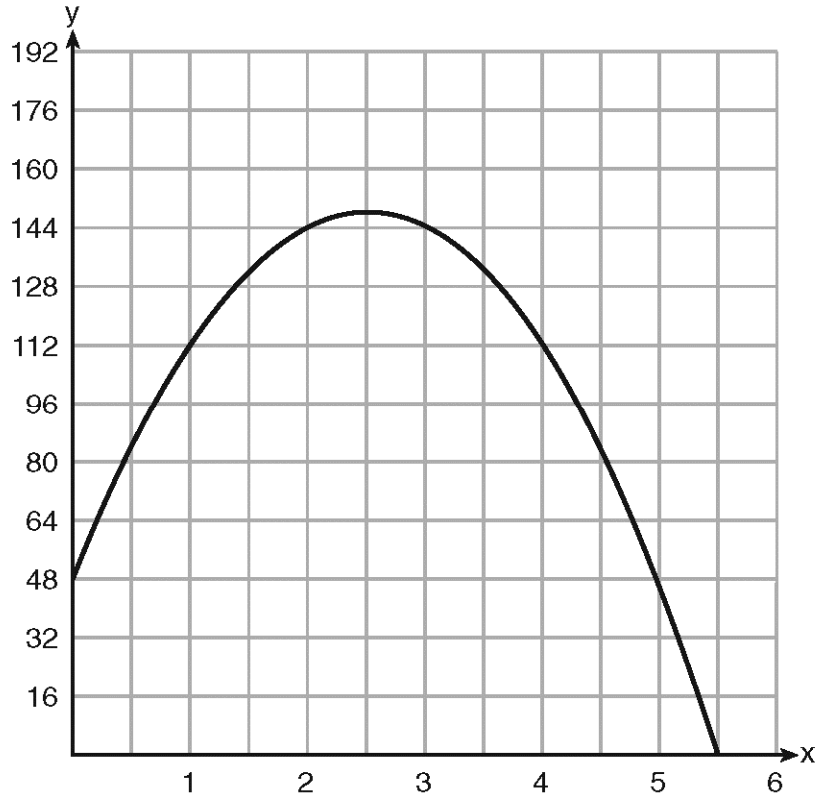
$$x^2 - 6x - 9 = 12 - 9$$

$$(x - 3)^2 = 3$$

Rationale: Choices (1), (3), and (4) are plausible but incorrect. They represent common student errors made when students use the method of completing the square to transform a given quadratic equation into an equation of the form $(x - p)^2 = q$. Compare with questions #27 and #33, which also assess A-REI.B.

#9

9 A ball is thrown into the air from the edge of a 48-foot-high cliff so that it eventually lands on the ground. The graph below shows the height, y , of the ball from the ground after x seconds.



For which interval is the ball's height always *decreasing*?

- (1) $0 \leq x \leq 2.5$ (3) $2.5 < x < 5.5$
(2) $0 < x < 5.5$ (4) $x \geq 2$

Measured CCLS Cluster: F-IF.B

Key: (3)

Commentary: This question measures the knowledge and skills described by the standards within F-IF.B because the student is required to interpret a key feature of a graph in terms of the quantities being modeled. Specifically, the student must interpret a graph to determine the interval for which a ball's height is decreasing. The item requires that the student employ MP.4 in relating a mathematical model to the situation being described and also MP.6 (Attend to precision) when examining the graph.

Answer Choice (1): $0 \leq x \leq 2.5$. This response is incorrect and represents an interval on which the ball is increasing in height. The student may have confused the concepts “increasing” and “decreasing” or misunderstood the way that the graph displays information about the quantities being modeled. A student who selects this response may have a limited understanding of how to interpret a key feature of a graph in terms of the quantities being modeled.

Answer Choice (2): $0 < x < 5.5$. This response is incorrect and represents the entire interval during which the ball is in the air. The student may have assumed that the ball’s height would decrease throughout the entire time period being modeled or misunderstood the way that the graph displays information about the quantities being modeled. A student who selects this response may have a limited understanding of how to interpret a key feature of a graph in terms of the quantities being modeled.

Answer Choice (3): $2.5 < x < 5.5$. This response is correct and represents an interval for which the ball’s height is decreasing. The graph shows that the ball’s height is decreasing from 2.5 seconds until 5.5 seconds after being thrown. A student who selects this response successfully interpreted a key feature of a graph in terms of the quantities being modeled.

Answer Choice (4): $x \geq 2$. This response is incorrect and represents an interval during which the height of the ball is both increasing and decreasing. The student may have imprecisely interpreted the features of the graph or misunderstood the way that the graph displays information about the quantities being modeled. A student who selects this response may have a limited understanding of how to interpret a key feature of a graph in terms of the quantities being modeled.

Rationale: Choices (1), (2), and (4) are plausible but incorrect. They represent common student errors made when interpreting a key feature of a graph in terms of the quantities being modeled. Choosing the correct response requires that students understand that the height of the ball is decreasing for the time after which it reaches its maximum height which is represented by the vertex of the graph. Compare with questions #2 and #18, which also assess F-IF.B.

#12

12 Keith determines the zeros of the function $f(x)$ to be -6 and 5 .
What could be Keith's function?

(1) $f(x) = (x + 5)(x + 6)$ (3) $f(x) = (x - 5)(x + 6)$

(2) $f(x) = (x + 5)(x - 6)$ (4) $f(x) = (x - 5)(x - 6)$

Measured CCLS Cluster: A-SSE.B

Key: (3)

Commentary: This question measures the knowledge and skills described by the standards within A-SSE.B because the student must identify the factored form of a quadratic function based upon its zeros. Zeros are input values that make a function equal to zero; in this case, they are values of x such that they make the equation $f(x) = 0$ true.

Answer Choice (1): $f(x) = (x + 5)(x + 6)$. This response is incorrect and is a function with zeros of -5 and -6 . While one of the zeros of this function, -6 , is named in the stem, the function will not equal zero when 5 is substituted for x . A student who selects this response may have a limited understanding of the relationship between a function and its zeros.

$$f(-6) = (-6 + 5)(-6 + 6) = 0$$

$$f(5) = (5 + 5)(5 + 6) = 110$$

Answer Choice (2): $f(x) = (x + 5)(x - 6)$. This response is incorrect and is a function with zeros of -5 and 6 . The function will not equal zero when either of the zeros named in the stem, -6 and 5 , is substituted for x . A student who selects this response may have limited understanding of the relationship between a function and its zeros.

$$f(-6) = (-6 + 5)(-6 - 6) = 12$$

$$f(5) = (5 + 5)(5 - 6) = -10$$

Answer Choice (3): $f(x) = (x - 5)(x + 6)$. This response is correct and is a function that has the given zeros; when $x = 5$ and $x = -6$, $f(x) = 0$. A student who selects this response understands the relationship between a function and its zeros.

$$f(-6) = (-6 - 5)(-6 + 6) = 0$$

$$f(5) = (5 - 5)(5 + 6) = 0$$

Answer Choice (4): $f(x) = (x - 5)(x - 6)$. This response is incorrect and is a function with zeros of 5 and 6. While one of the zeros of this function, 5, is named in the stem, the function will not equal zero when -6 is substituted for x . A student who selects this response may have a limited understanding of the relationship between a function and its zeros.

$$f(-6) = (-6 - 5)(-6 - 6) = 121$$

$$f(5) = (5 - 5)(5 - 6) = 0$$

Rationale: Choices (1), (2), and (4) are plausible but incorrect. They represent common student errors made when students are determining a function based on given zeros. Choosing the correct solution requires students to understand the relationship between a function and its zeros.

#14

14 Which system of equations has the same solution as the system below?

$$2x + 2y = 16$$

$$3x - y = 4$$

(1) $2x + 2y = 16$
 $6x - 2y = 4$

(3) $x + y = 16$
 $3x - y = 4$

(2) $2x + 2y = 16$
 $6x - 2y = 8$

(4) $6x + 6y = 48$
 $6x + 2y = 8$

Measured CCLS Cluster: A-REI.C

Key: (2)

Commentary: This question measures the knowledge and skills described by the standards within A-REI.C because the student must identify a system of equations that has the same solution set as the given system. Specifically, the student must understand that replacing one equation with a multiple of that equation results in a system with the same solution. In contrast to a question that requires students to follow a procedure to determine the solution only, this question relies on the student's conceptual understanding of how systems with the same solution are related to each other.

Answer Choice (1): $\begin{matrix} 2x + 2y = 16 \\ 6x - 2y = 4 \end{matrix}$. This response is incorrect and is a system that does not have the same solution as the given system. The student may have replaced the second equation with an equation that shows multiplication of only the left side by 2. A student who selects this response may not fully understand that the entire equation must be multiplied by a common factor in order to create a system of equations that has the same solutions as the given system.

Answer Choice (2): $\begin{matrix} 2x + 2y = 16 \\ 6x - 2y = 8 \end{matrix}$. This response is correct and is a system that has the same solution as the given system. The second equation has been replaced by a multiple of that equation by 2. A student who selects this response understands how to identify a system of equations that has the same solution as the given system.

Answer Choice (3): $\begin{matrix} x + y = 16 \\ 3x - y = 4 \end{matrix}$. This response is incorrect and is a system that does not have the same solution as the given system. The student may have replaced the first equation with an equation that shows multiplication of only the left side by $\frac{1}{2}$. A student who selects this response may not fully understand that the entire equation must be multiplied by a common factor in order to create a system of equations that has the same solutions as the given system.

Answer Choice (4):
$$\begin{array}{l} 6x + 6y = 48 \\ 6x + 2y = 8 \end{array}$$
 · This response is incorrect and is a system that does not have the same solution as the given system. The student may have recognized that each equation had been changed and correctly recognized that the first equation in this answer choice was the result of multiplying the first equation given by a factor of 3, but did not carefully examine the second equation and notice the change in sign from negative to positive. The second equation is not a multiple of the second equation given. A student who selects this response may have limited understanding of how to identify a system of equations that has the same solution as the given system.

Rationale: Choices (1), (3), and (4) are plausible but incorrect. They represent common student errors made when students are identifying a system of equations that has the same solution as a given system. Choosing the correct solution requires students to recognize when two different systems of equations have the same solution.

#15

15 The table below represents the function F .

x	3	4	6	7	8
$F(x)$	9	17	65	129	257

The equation that represents this function is

(1) $F(x) = 3^x$

(3) $F(x) = 2^x + 1$

(2) $F(x) = 3x$

(4) $F(x) = 2x + 3$

Measured CCLS Cluster: F-LE.A

Key: (3)

Commentary: This question measures the knowledge and skills described by the standards within F-LE.A because the student is required to construct an exponential function based on input-output pairs presented in a table.

Answer Choice (1): $F(x) = 3^x$. This response is incorrect and is not a function that represents the input-output pairs in the table. The student may have misinterpreted the first input-output pair of (3,9) as sufficient evidence that 3^x would be the expression for $F(x)$, mistaking the presence of 3 as the base of an exponential expression. A student who selects this response may have a limited understanding of how to construct an exponential function based on input-output pairs presented in a table.

Answer Choice (2): $F(x) = 3x$. This response is incorrect and is not a function that represents the input-output pairs in the table. The student may have misinterpreted the first input-output pair of (3,9) as sufficient evidence that $3x$ would be the expression for $F(x)$, since $3(3) = 9$. A student who selects this response may have a limited understanding of how to construct an exponential function based on input-output pairs presented in a table.

Answer Choice (3): $F(x) = 2^x + 1$. This response is correct and is a function that represents the input-output pairs in the table. The expression $2^x + 1$ accurately relates x and $F(x)$ for all values named in the table. A student who selects this response understands how to construct an exponential function based on input-output pairs presented in a table.

$$F(3) = 2^3 + 1 = 9$$

$$F(4) = 2^4 + 1 = 17$$

$$F(6) = 2^6 + 1 = 65$$

$$F(7) = 2^7 + 1 = 129$$

$$F(8) = 2^8 + 1 = 257$$

Answer Choice (4): $F(x) = 2x + 3$. This response is incorrect and is not a function that represents the input-output pairs in the table. The student may have misinterpreted the first input-output pair of (3,9) as sufficient evidence that $2x + 3$ would be the expression for $F(x)$, since $2(3) + 3 = 9$. Additionally, the student may have misinterpreted the slope of 2 in the linear equation as an appropriate model for the rate of change depicted in the table. A student who selects this response may have a limited understanding of how to construct an exponential function based on input-output pairs presented in a table.

Rationale: Choices (1), (2), and (4) are plausible but incorrect. They represent common student errors made when constructing an exponential function based on input-output pairs presented in a table. Choosing the correct solution requires students to understand how a table of input-output pairs is related to a function. Compare with questions #6 and #24, which also assess F-LE.A.

#17

17 If $f(x) = \frac{1}{3}x + 9$, which statement is always true?

(1) $f(x) < 0$

(3) If $x < 0$, then $f(x) < 0$.

(2) $f(x) > 0$

(4) If $x > 0$, then $f(x) > 0$.

Measured CCLS Cluster: F-IF.A

Key: (4)

Commentary: This question measures the knowledge and skills described by the standards within F-IF.A because the student is required to interpret statements that use function notation. Additionally, the item requires the student to employ MP.2 (Reason abstractly and quantitatively), as they must understand and interpret the meaning of an abstract representation.

Answer Choice (1): $f(x) < 0$. This response is incorrect and does not represent a true statement about the function, since $f(x)$ is not always negative. The student may have mistakenly assumed that $f(x)$ would always be negative since it is negative for some values of x . A student who selects this may have limited understanding of how to evaluate functions for inputs in their domains and interpret statements that use function notation.

Answer Choice (2): $f(x) > 0$. This response is incorrect and does not represent a true statement about the function, since $f(x)$ is not always positive. The student may have mistakenly assumed that $f(x)$ would always be positive since it is positive, for some values of x , and since the linear equation has a positive y -intercept and slope. A student who selects this may have a limited understanding of how to evaluate functions for inputs in their domains and interpret statements that use function notation.

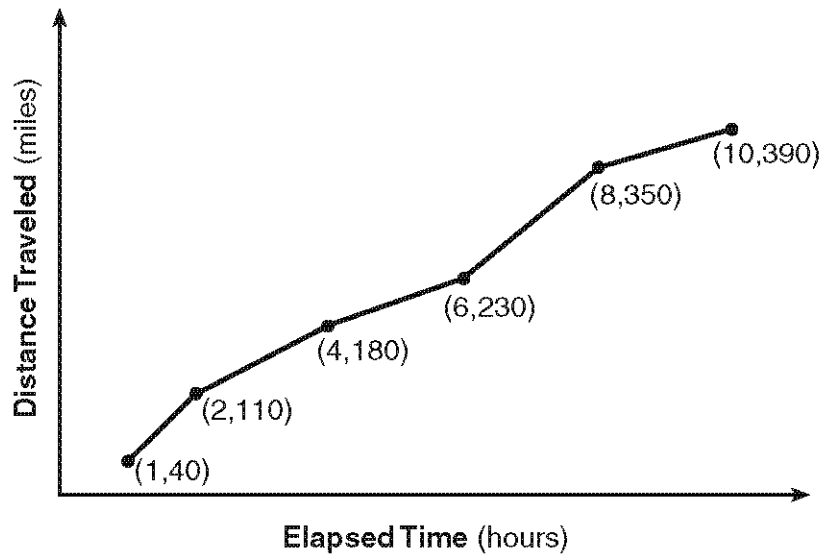
Answer Choice (3): If $x < 0$, then $f(x) < 0$. This response is incorrect and does not represent a true statement about the function, since $f(x)$ is not always negative when $x < 0$. The student may have mistakenly assumed that $f(x)$ would be negative when x is negative since this is true in some cases (for example, when $x = -30$). A student who selects this may have a limited understanding of how to evaluate functions for inputs in their domains and interpret statements that use function notation.

Answer Choice (4): If $x > 0$, then $f(x) > 0$. This response is correct and represents a true statement about the function, since $f(x)$ will be positive for all values of x greater than 0. A student who selects this response understands how to evaluate functions for inputs in their domains and interpret statements that use function notation.

Rationale: Choices (1), (2), and (3) are plausible but incorrect. They represent common student errors made when evaluating functions for inputs in their domains. Choosing the correct solution requires the student to interpret statements that use function notation. Compare with questions #20, #21, and #30, which also assess F-IF.A.

#18

18 The Jamison family kept a log of the distance they traveled during a trip, as represented by the graph below.



During which interval was their average speed the greatest?

- (1) the first hour to the second hour
- (2) the second hour to the fourth hour
- (3) the sixth hour to the eighth hour
- (4) the eighth hour to the tenth hour

Measured CCLS Cluster: F-IF.B

Key: (1)

Commentary: This question measures the knowledge and skills described by the standards within F-IF.B because the student is required to calculate and interpret the rate of change of a function in a real-world context. In this context, the student must interpret the rate of change as the average speed of the Jamison family. This question also signals MP.6 (Attend to precision), as the student will need to carefully read and interpret values from the graph.

Answer Choice (1): The first hour to the second hour. This response is correct and represents the interval with the highest average speed which is 70 miles per hour. A student who selects this response understands how to calculate and interpret the rate of change of a function in a real-world context.

Answer Choice (2): The second hour to the fourth hour. This response is incorrect and does not represent the interval with the highest average speed. The student may have mistakenly interpreted the fact that the distance traveled (70 miles) is greater for this interval, than for the interval from the first hour to the second hour, as evidence that the average speed is greatest on this interval. A student who selects this response has a limited understanding of how to calculate and interpret the rate of change of a function in a real-world context.

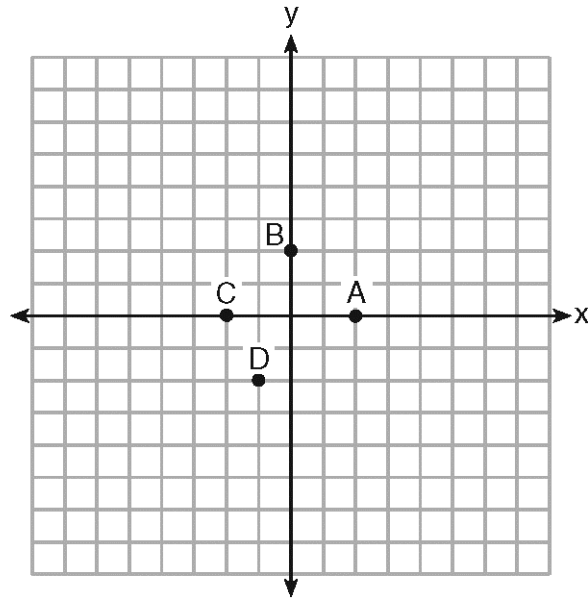
Answer Choice (3): The sixth hour to the eighth hour. This response is incorrect and does not represent the interval with the highest average speed. The student may have mistakenly interpreted the fact that the distance traveled (120 miles) is greater than any other interval as evidence that the average speed is greatest on this interval. Alternatively, the student may also have determined that the slope over that interval was the steepest by inspection without carefully comparing it to the steeper slope of the interval from the first to the second hour. A student who selects this response may lack precision and care in determining the solution and has a limited understanding of how to calculate and interpret the rate of change of a function in a real-world context.

Answer Choice (4): The eighth hour to the tenth hour. This response is incorrect and does not represent the interval with the highest average speed. The student may have mistakenly interpreted the fact that the total distance traveled (390 miles) by the end of this interval is greater than at the end of any other interval as evidence that the average speed is greatest on this interval. Alternatively, the student may have mistakenly interpreted the interval with the smallest value for slope as an indication of the greatest average speed. A student who selects this response has a limited understanding of how to calculate and interpret the rate of change of a function in a real-world context.

Rationale: Choices (2), (3), and (4) are plausible but incorrect. They represent common student errors made when calculating and interpreting the rate of change of a function in a real-world context. Choosing the correct solution requires students to calculate the rate of change and interpret it as the average speed of the Jamison family. Compare with questions #2 and #9, which also assess F-IF.B.

#20

20 The graph of $y = f(x)$ is shown below.



Which point could be used to find $f(2)$?

- | | |
|-------|-------|
| (1) A | (3) C |
| (2) B | (4) D |

Measured CCLS Cluster: F-IF.A

Key: (1)

Commentary: This question measures the knowledge and skills described by the standards within F-IF.A because students are required to evaluate a function for inputs in its domain and understand that the graph of a function f is the graph of the equation $y = f(x)$. Specifically, the student must identify a point on the graph of f that could be used to determine $f(2)$.

Answer Choice (1): A. This response is correct and represents the point that could be used to determine $f(2)$. The student correctly determined that $f(2)$ would be represented by the point $(2,0)$ since the x -value is 2. A student who selects this response understands how to evaluate a function for inputs in its domain and that the graph of a function f is the graph of the equation $y = f(x)$.

Answer Choice (2): B. This response is incorrect and represents a point that cannot be used to find $f(2)$. The student may have mistakenly assumed that the value of 2 in $f(2)$ represented the y -value rather than the x -value. A student who selects this response may have a limited understanding of how to evaluate a function for inputs in its domain and that the graph of a function f is the graph of the equation $y = f(x)$.

Answer Choice (3): C. This response is incorrect and represents a point that cannot be used to find $f(2)$. The student may have mistakenly assumed that $f(-2)$ would have the same value as $f(2)$ or confused the positive and negative directions on the x -axis. A student who selects this response may have a limited understanding of how to evaluate a function for inputs in its domain and that the graph of a function f is the graph of the equation $y = f(x)$.

Answer Choice (4): D. This response is incorrect and represents a point that cannot be used to find $f(2)$. The student may have mistakenly assumed that the value of 2 in $f(2)$ represented the y -value rather than the x -value and then also confused the positive and negative directions on the y -axis. A student who selects this response may have a limited understanding of how to evaluate a function for inputs in its domain and that the graph of a function f is the graph of the equation $y = f(x)$.

Rationale: Choices (2), (3), and (4) are plausible but incorrect. They represent common student errors made when identifying a point on the graph of f that could be used to determine $f(2)$. Choosing the correct solution requires students to understand that the graph of a function f is the graph of the equation $y = f(x)$. Compare with questions #17, #21, and #30, which also assess F-IF.A.

#21

21 A sunflower is 3 inches tall at week 0 and grows 2 inches each week. Which function(s) shown below can be used to determine the height, $f(n)$, of the sunflower in n weeks?

I. $f(n) = 2n + 3$

II. $f(n) = 2n + 3(n - 1)$

III. $f(n) = f(n - 1) + 2$ where $f(0) = 3$

(1) I and II

(3) III, only

(2) II, only

(4) I and III

Measured CCLS Cluster: F-IF.A

Key: (4)

Commentary: This question measures the knowledge and skills described by the standards within F-IF.A because the student is required to recognize that a sequence is a function, sometimes defined recursively. Specifically, the student must identify different, equally valid representations of a sequence that models the growth of a flower; the function named in “III” is a sequence that is defined recursively. In identifying the function, the student must employ MP.4 to determine the validity of a series of models.

Answer Choice (1): I and II. This response is incorrect because while “I” can be used to determine the height of the flower after n weeks, “II” cannot. The student may have assumed incorrectly that since the flower grows 2 inches each week, all expressions containing $2n$ would be valid for $f(n)$. The student may not have recognized that option “III” is another possible representation of this function defined recursively rather than explicitly. A student who selects this response may have a limited understanding that a sequence can be defined by a function, both explicitly and recursively.

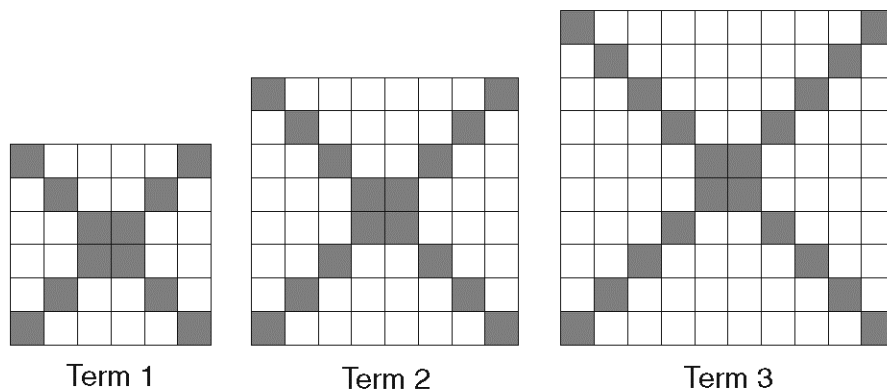
Answer Choice (2): II only. This response is incorrect because “II” cannot be used to determine the height of the flower after n weeks. The student may have assumed incorrectly that since the flower grows 2 inches each week, this expression would be valid for $f(n)$ since it contains $2n$. The student may also have made a procedural error when performing computations to validate this response or may not have recognized that option “III” is another possible representation of this function, defined recursively rather than explicitly. A student who selects this response may have a limited understanding that a sequence is a function, sometimes defined recursively.

Answer Choice (3): III only. This response is incorrect because “III” is not the only function that can be used to determine the height of the flower after n weeks. The student may have assumed that the function must be defined recursively or made a procedural error when examining the function named in “I”. A student who selects this response may have a limited understanding that a sequence can be defined by a function, both explicitly and recursively.

Answer Choice (4): I and III. This response is correct because it names both functions that can be used to determine the height of the flower after n weeks. A student who selects this response understands that a sequence can be defined by a function, both explicitly and recursively.

Rationale: Choices (1), (2), and (3) are plausible but incorrect. They represent common student errors made when analyzing different functions that model a situation. Choosing the correct solution requires students to recognize that a function can be defined explicitly and recursively. Compare with questions #17, #20, and #31, which also assess F-IF.A.

24 The diagrams below represent the first three terms of a sequence.



Assuming the pattern continues, which formula determines a_n , the number of shaded squares in the n th term?

- (1) $a_n = 4n + 12$ (3) $a_n = 4n + 4$
 (2) $a_n = 4n + 8$ (4) $a_n = 4n + 2$

Measured CCLS Cluster: F-LE.A

Key: (2)

Commentary: This question measures the knowledge and skills described by the standards within F-LE.A because the student is required to construct an arithmetic sequence. In this case, the sequence is based on a relationship described by a visual pattern. The pattern and description of the sequence indicate that a_n must give the number of shaded squares in “Term n ” or the n th term.

Answer Choice (1): $a_n = 4n + 12$. This response is incorrect because the sequence described by the formula $a_n = 4n + 12$ does not determine the number of shaded squares in the n th term. The student may have assumed this formula would be valid since each “term” has a central area of four shaded squares and the second term has 12 shaded squares in addition to the central area of four. A student who selects this response may have a limited understanding of how to construct an arithmetic sequence from a relationship described by a visual pattern.

Answer Choice (2): $a_n = 4n + 8$. This response is correct because the sequence described by the formula $a_n = 4n + 8$ determines the number of shaded squares in the n th term. A student who selects this response understands how to construct an arithmetic sequence from a relationship described by a visual pattern.

$$a_1 = 4(1) + 8 = 12 \text{ shaded squares in Term 1}$$

$$a_2 = 4(2) + 8 = 16 \text{ shaded squares in Term 2}$$

$$a_3 = 4(3) + 8 = 20 \text{ shaded squares in Term 3}$$

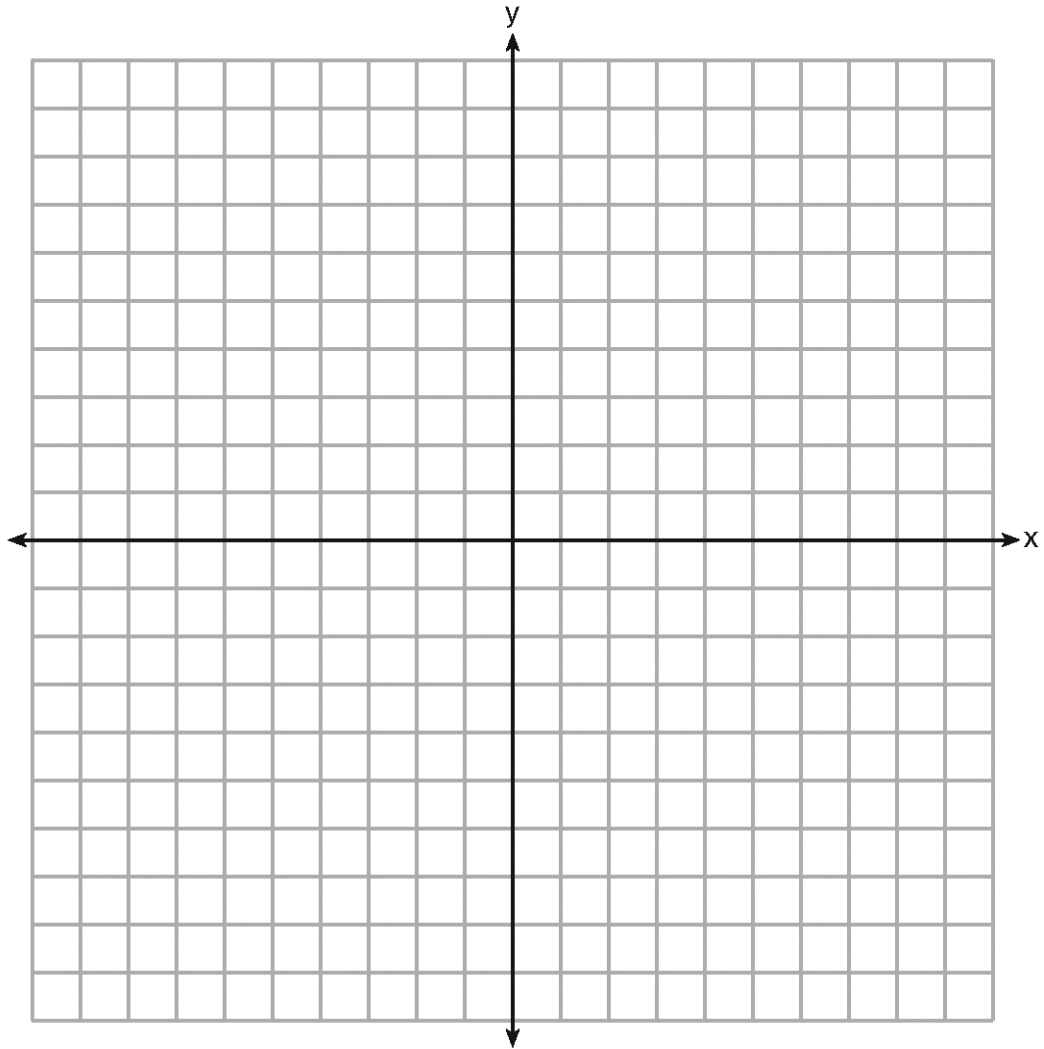
Answer Choice (3): $a_n = 4n + 4$. This response is incorrect because the sequence described by the formula $a_n = 4n + 4$ does not determine the number of shaded squares in the n th term. The student may have assumed this formula would be valid since each “term” has a central area of four shaded squares and each term adds four additional shaded blocks. A student who selects this response may have a limited understanding of how to construct an arithmetic sequence from a relationship described by a visual pattern.

Answer Choice (4): $a_n = 4n + 2$. This response is incorrect because the sequence described by the formula $a_n = 4n + 2$ does not determine the number of shaded squares in the n th term. The student may have assumed this formula would be valid since each “term” has a central area of four shaded squares and each term adds two more rows and two more columns to the prior term. A student who selects this response may have a limited understanding of how to construct an arithmetic sequence from a relationship described by a visual pattern.

Rationale: Choices (1), (3), and (4) are plausible but incorrect. They represent common student errors made in determining the formula that describes an arithmetic sequence represented visually. Compare with questions #6 and #15, which also assess F-LE.A.

#25

25 Draw the graph of $y = \sqrt{x} - 1$ on the set of axes below.



Measured CCLS Cluster: F-IF.C

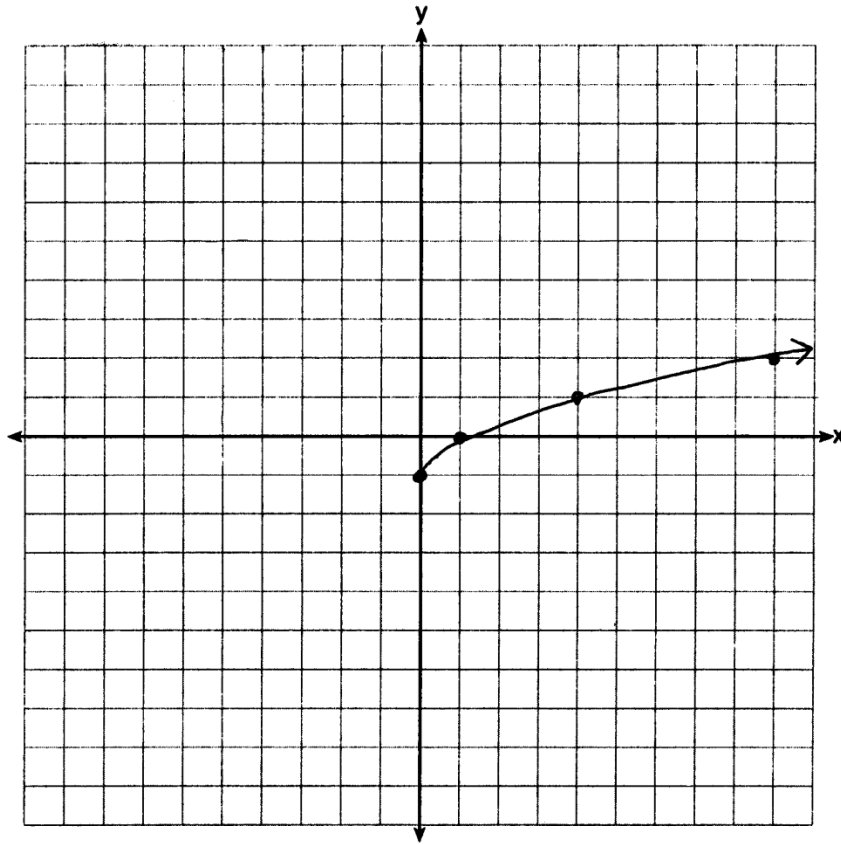
Commentary: The question measures the knowledge and skills described by the standards within F-IF.C because it requires the student to graph a square root function.

Rationale: This question requires students to draw the graph of the equation $y = \sqrt{x} - 1$. As indicated in the rubric, a correct response requires a correct graph to be drawn. A correct graph of the equation $y = \sqrt{x} - 1$ would include an endpoint at $(0, -1)$. The determining factor in demonstrating a thorough understanding is using mathematically sound procedures that lead the student to create a correct graph.

Sample student responses and scores appear on the following pages.

Question 25

25 Draw the graph of $y = \sqrt{x} - 1$ on the set of axes below.

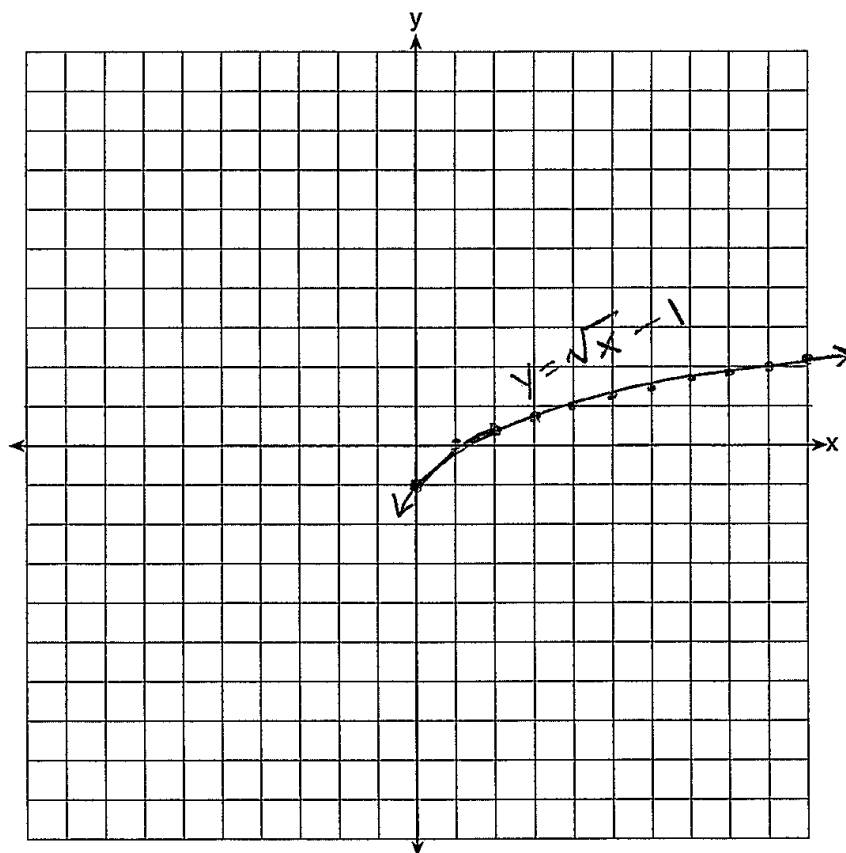


x	y
0	-1
1	0
4	1
9	2

Score 2: The student has a complete and correct response.

Question 25

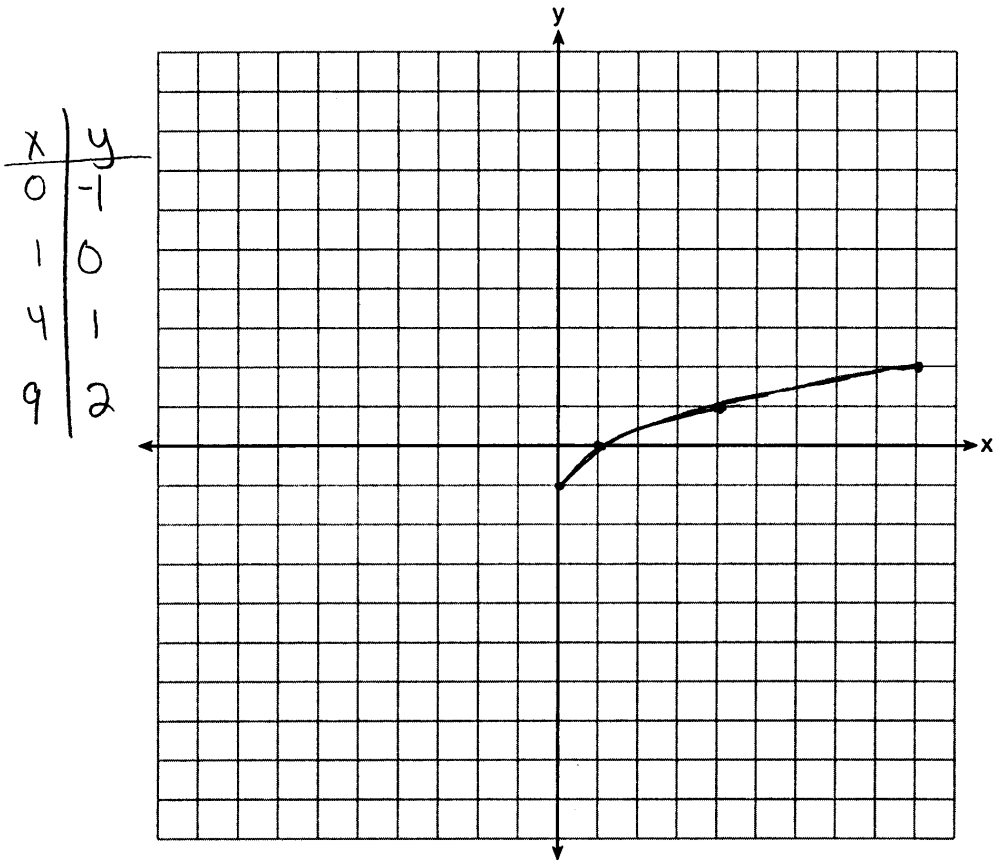
25 Draw the graph of $y = \sqrt{x} - 1$ on the set of axes below.



Score 1: The student made one error by extending the graph beyond the point $(0, -1)$.

Question 25

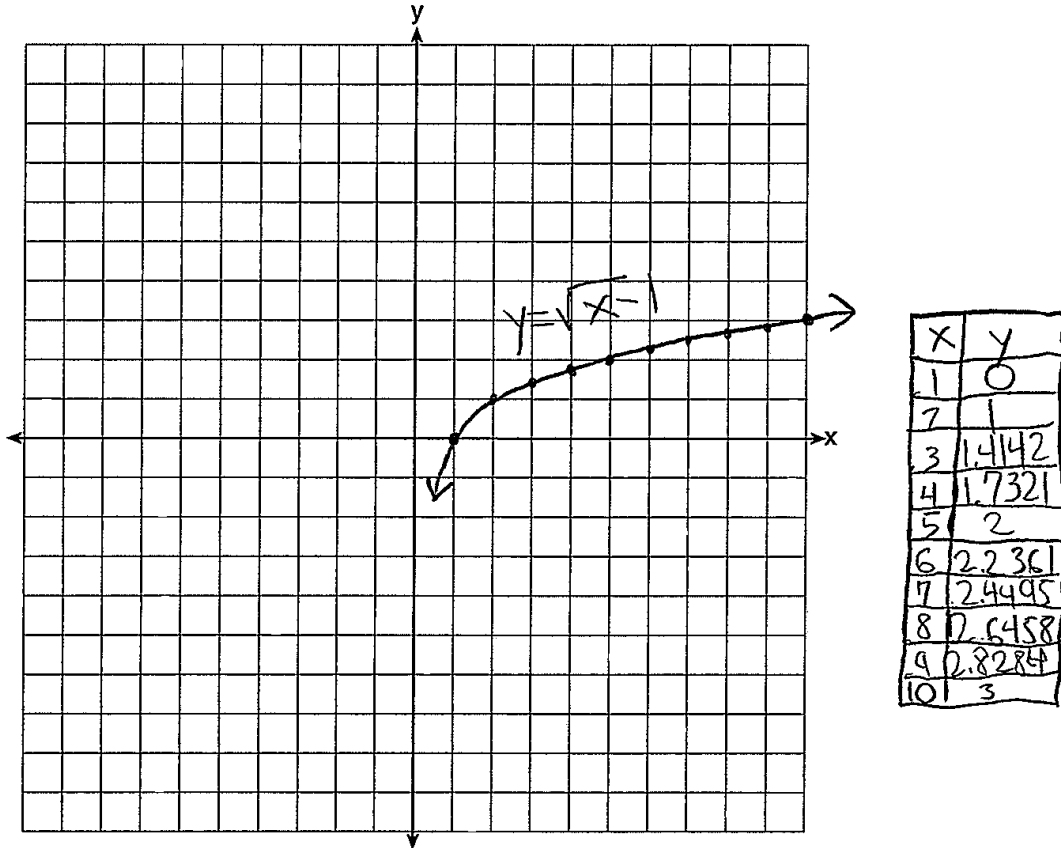
25 Draw the graph of $y = \sqrt{x} - 1$ on the set of axes below.



Score 1: The student made one graphing error by not extending the graph beyond the point (9,2).

Question 25

25 Draw the graph of $y = \sqrt{x} - 1$ on the set of axes below.



Score 0: The student made two errors by graphing $y = \sqrt{x-1}$ and by extending the graph beyond the point (1,0).

#26:

26 The breakdown of a sample of a chemical compound is represented by the function $p(t) = 300(0.5)^t$, where $p(t)$ represents the number of milligrams of the substance and t represents the time, in years. In the function $p(t)$, explain what 0.5 and 300 represent.

Measured CCLS Cluster: F-LE.B

Commentary: The question measures the knowledge and skills described by the standards within F-LE.B because the student is required to interpret the parameters of an exponential function within a given real-world context. The question also signals MP.2 (Reason abstractly and quantitatively), as the student is required to interpret the contextualized meaning of parts of an algebraic expression.

Rationale: This question asks students to identify and explain the parameters of an exponential decay function where the inputs and outputs have been defined. Generally, for decay functions of the form $f(t) = Ab^t$, A represents the “initial value,” and b represents the quantity $1 - r$, where r is the percent of decay over each time interval, sometimes called the “rate of decay;” b is sometimes called the “decay factor.” In the case of this function, $r = 0.5$ and consequently, $b = 0.5$, so 0.5 can be thought of as representing the decay factor or the rate of decay. As indicated in the rubric, a correct response requires that correct explanations are made, such as 0.5 is the rate of decay and 300 is the initial amount. The determining factor in demonstrating a thorough explanation is that the student clearly understands the situation modeled by the function and what the numerical values represent.

Sample student responses and scores appear on the following pages.

Question 26

26 The breakdown of a sample of a chemical compound is represented by the function $p(t) = 300(0.5)^t$, where $p(t)$ represents the number of milligrams of the substance and t represents the time, in years. In the function $p(t)$, explain what 0.5 and 300 represent.

$$p(t) = 300(0.5)^t$$

↑
time

↑
milligrams

300 represents the original amount of the chemical that was to begin with and .5 represents the rate of decay

Score 2: The student has a complete and correct response.

Question 26

26 The breakdown of a sample of a chemical compound is represented by the function $p(t) = 300(0.5)^t$, where $p(t)$ represents the number of milligrams of the substance and t represents the time, in years. In the function $p(t)$, explain what 0.5 and 300 represent.

(300) is the number at which
It starts at

(.5) is the percent or rate the
Number (300) goes up by.

Score 1: The student gave a correct explanation for 300, but the explanation for 0.5 is incorrect.

Question 26

26 The breakdown of a sample of a chemical compound is represented by the function $p(t) = 300(0.5)^t$, where $p(t)$ represents the number of milligrams of the substance and t represents the time, in years. In the function $p(t)$, explain what 0.5 and 300 represent.

t = time in years

p = milligrams

0.5 represents the
milligrams and 300
represents the time in
years

Score 0: The student's response was completely incorrect.

#27:

27 Given $2x + ax - 7 > -12$, determine the largest integer value of a when $x = -1$.

Measured CCLS Cluster: A-REI.B

Commentary: The question measures the knowledge and skills described by the standards within A-REI.B because the student is required to solve a linear inequality in one variable that includes coefficients represented by letters.

Rationale: This question asks the student to solve a given linear inequality for a variable and determine the largest possible integer value for the variable that will make the inequality true. As indicated in the rubric, a correct response will include 2 with correct work shown including a set of computations leading to a correct response that the largest integer is 2.

The determining factor in demonstrating a thorough understanding is using mathematically sound procedures to lead to a correct response. Partially correct solutions may include an inequality expressing that $a < 3$ or an inequality that has not been solved properly.

The correct answer may be arrived at by applying properties of operations to solve the given linear inequality for the variable:

$$2x + ax - 7 > -12$$

$$2(-1) + a(-1) - 7 > -12$$

$$-2 + (-a) - 7 > -12$$

$$-a - 9 > -12$$

$$-a > -3$$

$$a < 3$$

The student would then interpret the statement $a < 3$, naming 2 as the largest possible integer value of a .

Compare with questions #8 and #33, which also assess A-REI.B.

Sample student responses and scores appear on the following pages.

Question 27

27 Given $2x + ax - 7 > -12$, determine the largest integer value of a when $x = -1$.

$$x = -1$$

$$2(-1) + a(-1) - 7 > -12$$

$$\textcircled{-2} + -1a \textcircled{-7} > -12$$

$$\begin{array}{r} -9 \\ +9 \\ \hline \end{array} \quad -1a > -12$$

$$a = 2$$

$$\begin{array}{r} -a > -3 \\ -1 & -1 \\ \hline \end{array}$$

$$a < 3$$

Check

$$2(-1) + (-1)(2) - 7 > -12$$

$$\textcircled{-2} \quad \textcircled{-2} \textcircled{-7} > -12$$

$$\boxed{-11 > -12}$$

Score 2: The student has a complete and correct response.

Question 27

27 Given $2x + ax - 7 > -12$, determine the largest integer value of a when $x = -1$.

$$x = -1$$

$$2x + ax - 7 > -12$$

$$2(-1) + a(-1) - 7 > -12$$

$$\textcircled{-2} + \textcircled{-a} \textcircled{-7} > -12$$

$$\begin{array}{r} -9 + -a > -12 \\ +9 \quad \quad +9 \end{array}$$

$$\begin{array}{r} -2 = -3 \\ \hline -1 = -1 \\ \boxed{a = 3} \end{array}$$

Score 1: The student made one error by not following through with the inequality sign.

Question 27

27 Given $2x + ax - 7 > -12$, determine the largest integer value of a when $x = -1$.

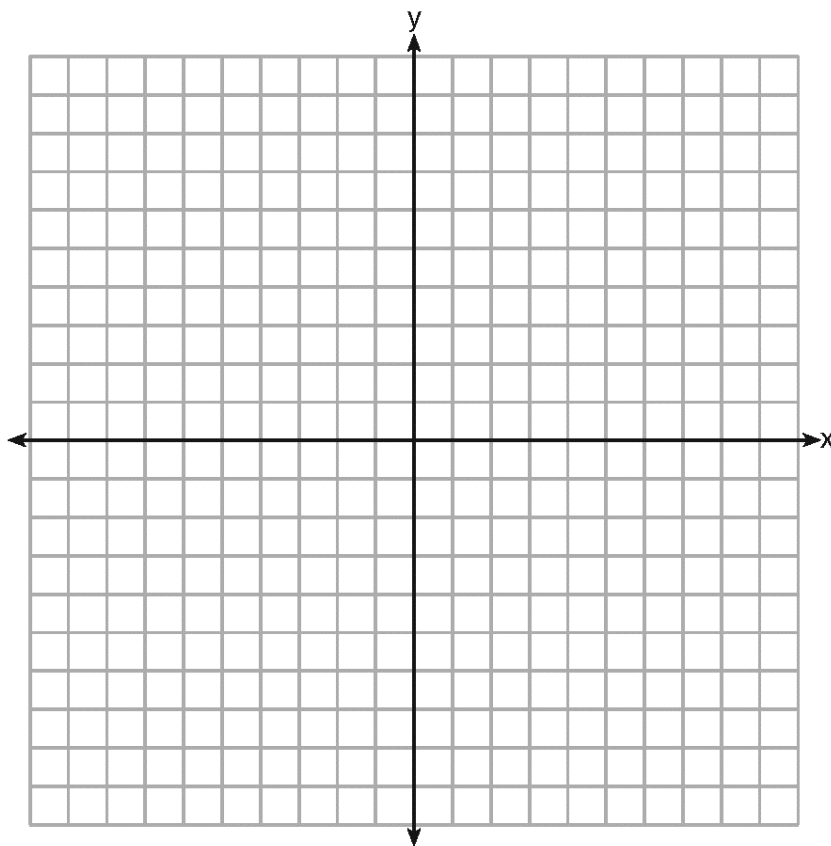
$$\begin{aligned} 2(-1) + a(-1) - 7 &> -12 \\ -\cancel{2} + -1a - 7 &> -12 \\ +\cancel{2} \quad \quad \quad +2 & \\ a - \cancel{5} &> -12 \\ +\cancel{5} \quad \quad +5 & \\ \hline \boxed{-a > -7} & \end{aligned}$$

Score 0: The student made two conceptual errors by adding 2 to the same side of the inequality twice and not stating the largest integer.

#28:

- 28 The vertex of the parabola represented by $f(x) = x^2 - 4x + 3$ has coordinates $(2, -1)$. Find the coordinates of the vertex of the parabola defined by $g(x) = f(x - 2)$. Explain how you arrived at your answer.

[The use of the set of axes below is optional.]



Measured CCLS Cluster: F-BF.B

Commentary: The question measures the knowledge and skills described by the standards within F-BF.B because the student is required to identify the effect on the vertex of the parabola given by f , when $f(x)$ is replaced with $f(x + k)$. Additionally, the question requires the student to employ MP.3 (Construct viable arguments and critique the reasoning of others), as they must provide an explanation describing how they determined this effect.

Rationale: This question requires the student to determine the coordinates of the vertex of the parabola defined by $g(x) = f(x - 2)$ and also provide a correct explanation describing the procedure or strategy used to find the new vertex. As indicated in the rubric, a correct response includes the new vertex $(4, -1)$ and a correct explanation. The explanation should describe in words the procedure or strategy used to determine the new vertex. The determining factor in demonstrating a thorough understanding is using mathematically sound procedures to lead to a correct response.

The correct answer may be found by graphing the function, $f(x) = x^2 - 4x + 3$ then drawing a new graph for $g(x) = f(x - 2)$ that is shifted two units right which shows the new vertex at $(4, -1)$. Additionally, an appropriate explanation is included that describes the shift of the initial graph, $f(x)$. The explanation may include a description that replacing $f(x)$ with $f(x + k)$ has the effect of a horizontal shift k units to the left; correspondingly, replacing $f(x)$ with $f(x - 2)$ has the effect of a horizontal shift 2 units to the right.

Additionally, the correct answer may be found algebraically by substituting the expression $(x - 2)$ for x into the expression for $f(x)$ and examining its graph: $f(x - 2) = (x - 2)^2 - 4(x - 2) + 3$.

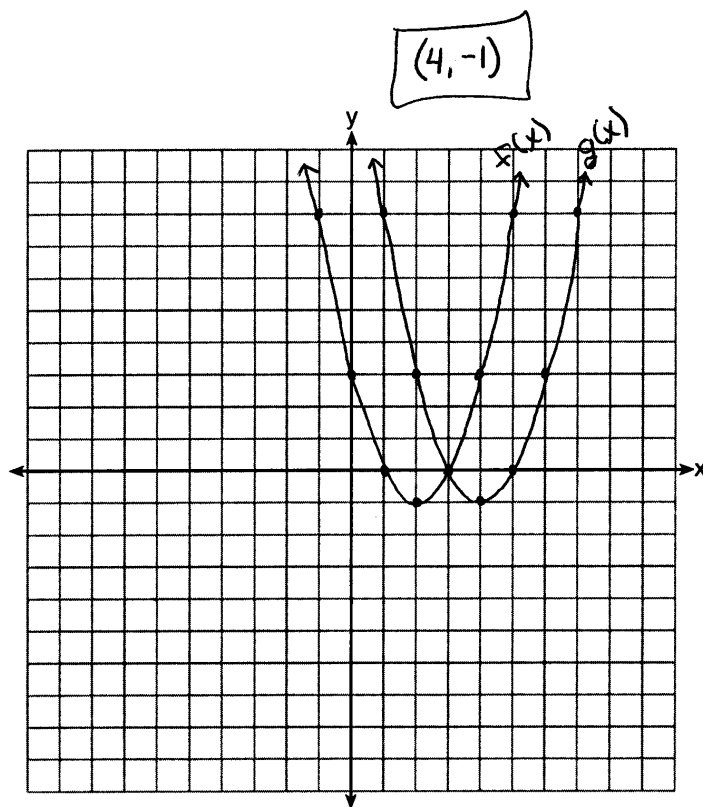
Sample student responses and scores appear on the following pages.

Question 28

28 The vertex of the parabola represented by $f(x) = x^2 - 4x + 3$ has coordinates $(2, -1)$. Find the coordinates of the vertex of the parabola defined by $g(x) = f(x - 2)$. Explain how you arrived at your answer.

[The use of the set of axes below is optional.]

I graphed $f(x)$ then $g(x)$ which is $(x-2)^2 - 4(x-2) + 3$



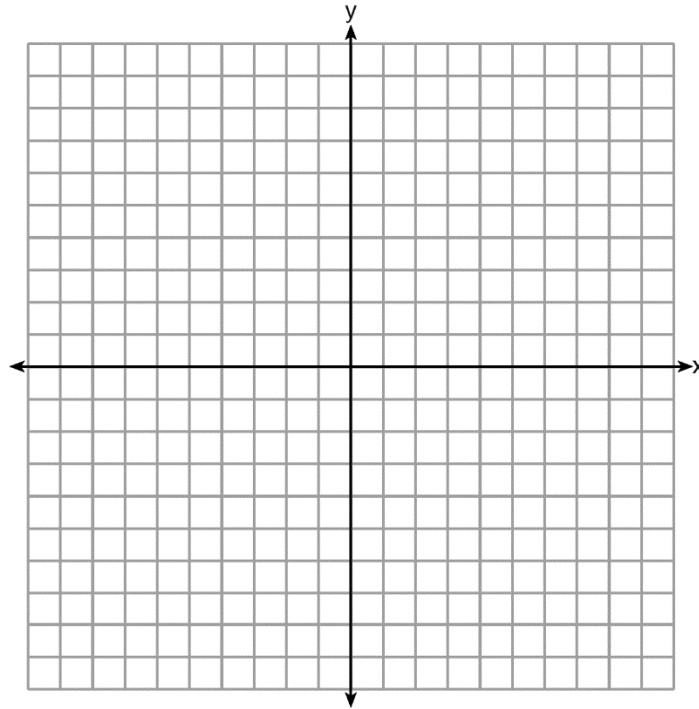
Score 2: The student has a complete and correct response.

Question 28

28 The vertex of the parabola represented by $f(x) = x^2 - 4x + 3$ has coordinates $(2, -1)$. Find the coordinates of the vertex of the parabola defined by $g(x) = f(x - 2)$. Explain how you arrived at your answer.

[The use of the set of axes below is optional.]

$(4, -1)$ $g(x)$ is a shift of
2 units right



Score 2: The student has a complete and correct response.

Question 28

28 The vertex of the parabola represented by $f(x) = x^2 - 4x + 3$ has coordinates $(2, -1)$. Find the coordinates of the vertex of the parabola defined by $g(x) = f(x - 2)$. Explain how you arrived at your answer.

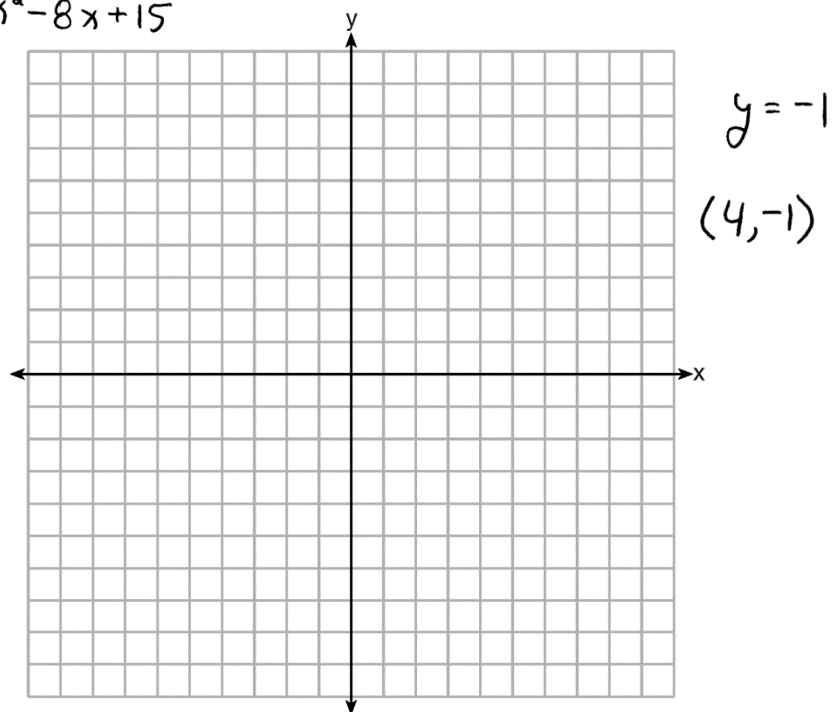
[The use of the set of axes below is optional.]

$$(x-2)^2 - 4(x-2) + 3$$

$$x^2 - 4x + 4 - 4x + 8 + 3$$

$$x^2 - 8x + 15$$

$$x = \frac{8}{2(1)} \rightarrow 4$$



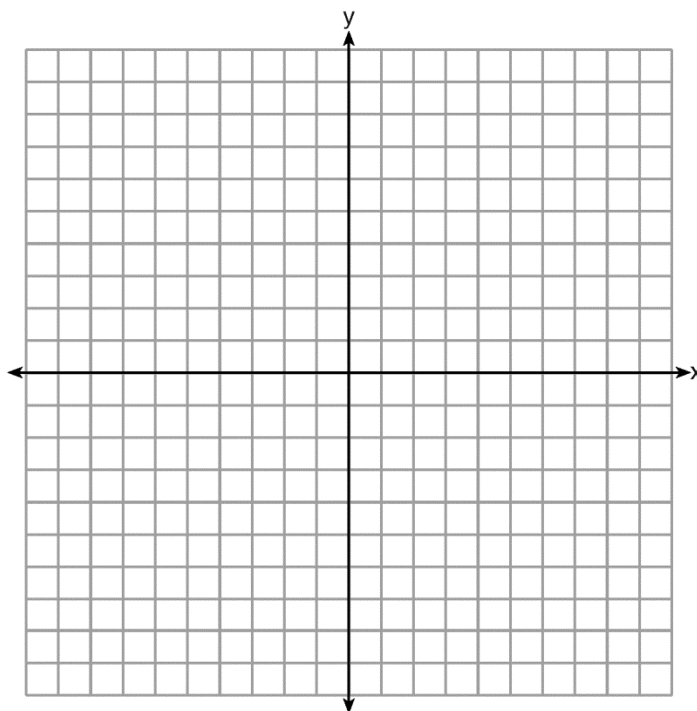
Score 1: The student did not provide an explanation.

Question 28

28 The vertex of the parabola represented by $f(x) = x^2 - 4x + 3$ has coordinates $(2, -1)$. Find the coordinates of the vertex of the parabola defined by $g(x) = f(x - 2)$. Explain how you arrived at your answer.

[The use of the set of axes below is optional.]

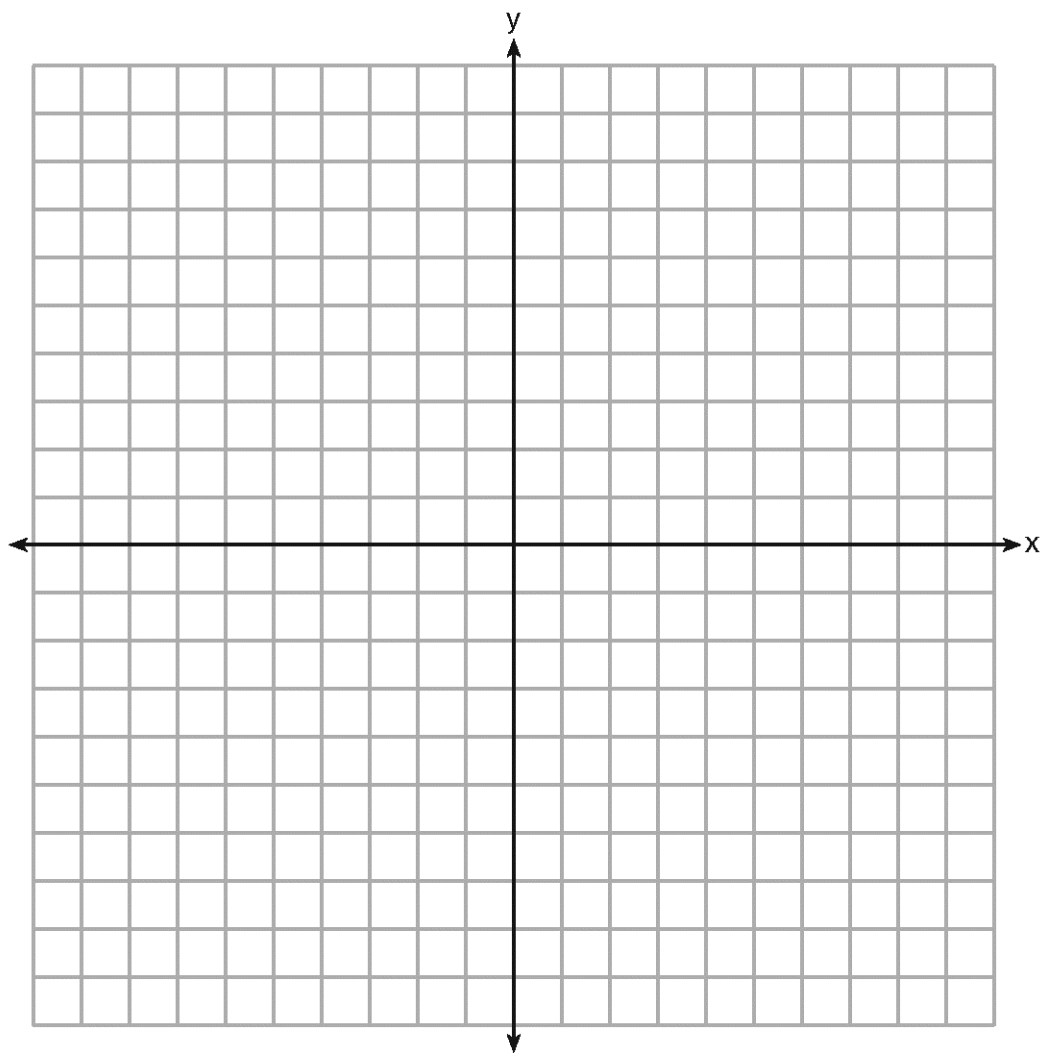
New Vertex $(4, 1)$ $(2, -1)$
shift of two units $+2 +2$



Score 0: The student made one conceptual error by adding 2 to both the x - and y -values of the vertex, and the explanation is incomplete because the direction is not indicated.

#29

29 On the set of axes below, draw the graph of the equation $y = -\frac{3}{4}x + 3$.



Is the point $(3,2)$ a solution to the equation? Explain your answer based on the graph drawn.

Measured CCLS Cluster: A-REI.D

Commentary: The question measures the knowledge and skills described by the standards within A-REI.D because it requires the student to understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane. In contrast to an item that assesses the procedural skill of drawing a graph only, this question requires students to reason about the meaning of the graph of an equation as the set of all solutions to the equation. Additionally, the student must employ MP.3, in providing evidence to support the claim about the point (3,2).

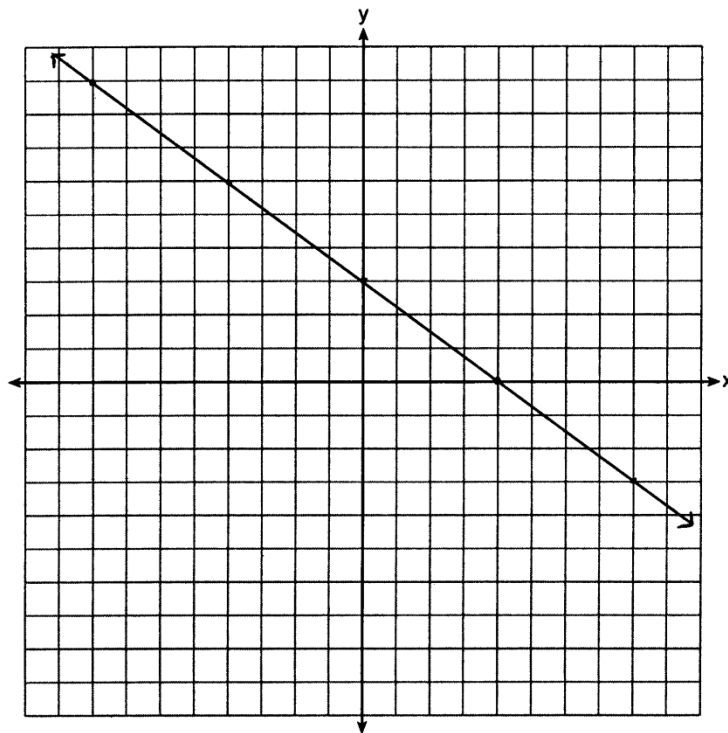
Rationale: This question requires students to graph a linear equation $y = -\frac{3}{4}x + 3$ then to determine whether the point (3,2) is a solution to the equation based on the graph. The graph of an equation is the set of all its solutions plotted in the coordinate plane; graphing the equation allows the student to evaluate whether or not the given point is a solution to the equation. As indicated in the rubric, a correct response includes a correct graph, “no,” and a correct explanation that is based on the graph. The explanation must indicate that the given point, (3,2), is *not* a solution to the given equation, since it is not part of the graphed solution set. The determining factor in demonstrating a thorough explanation includes providing evidence that the point does not fall on the line and therefore is not a solution to the equation.

Compare with question #37, which also assesses A-REI.D.

Sample student responses and scores appear on the following pages.

Question 29

29 On the set of axes below, draw the graph of the equation $y = -\frac{3}{4}x + 3$.



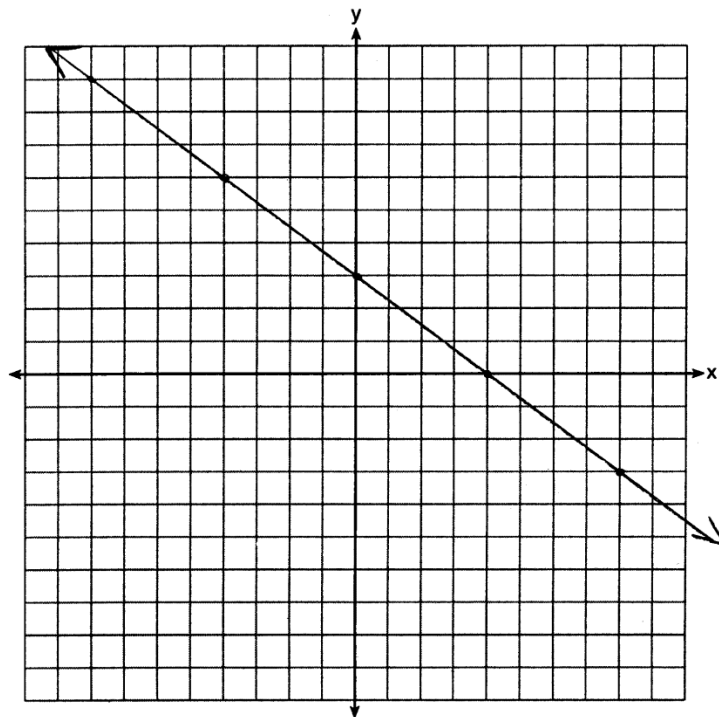
Is the point (3,2) a solution to the equation? Explain your answer based on the graph drawn.

No, it doesn't fall on the graph

Score 2: The student has a complete and correct response.

Question 29

29 On the set of axes below, draw the graph of the equation $y = -\frac{3}{4}x + 3$.



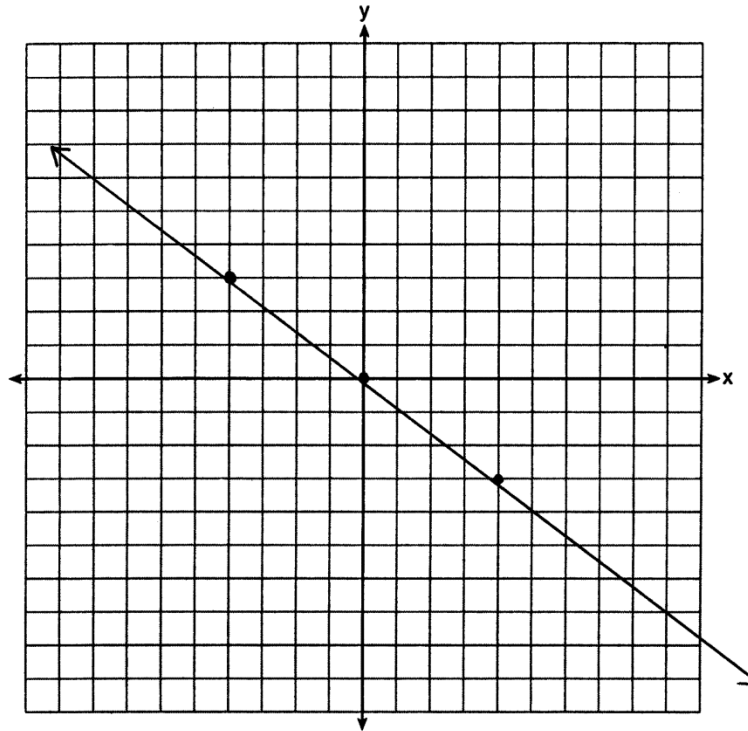
Is the point (3,2) a solution to the equation? Explain your answer based on the graph drawn.

$$\begin{aligned} -\frac{3}{4}(3) + 3 &= 2 \\ -2.25 + 3 &= 2 \\ .75 &\neq 2 \quad \text{No, it's not a solution} \end{aligned}$$

Score 1: A correct graph is drawn, but the explanation is not based on the graph.

Question 29

29 On the set of axes below, draw the graph of the equation $y = -\frac{3}{4}x + 3$.



Is the point (3,2) a solution to the equation? Explain your answer based on the graph drawn.

$$2 = -\frac{3}{4} \cdot 3 + 3$$

$$2 \stackrel{?}{=} -\frac{9}{4} + 3$$

$$2 \stackrel{?}{=} \frac{-27}{12} + \frac{36}{12}$$

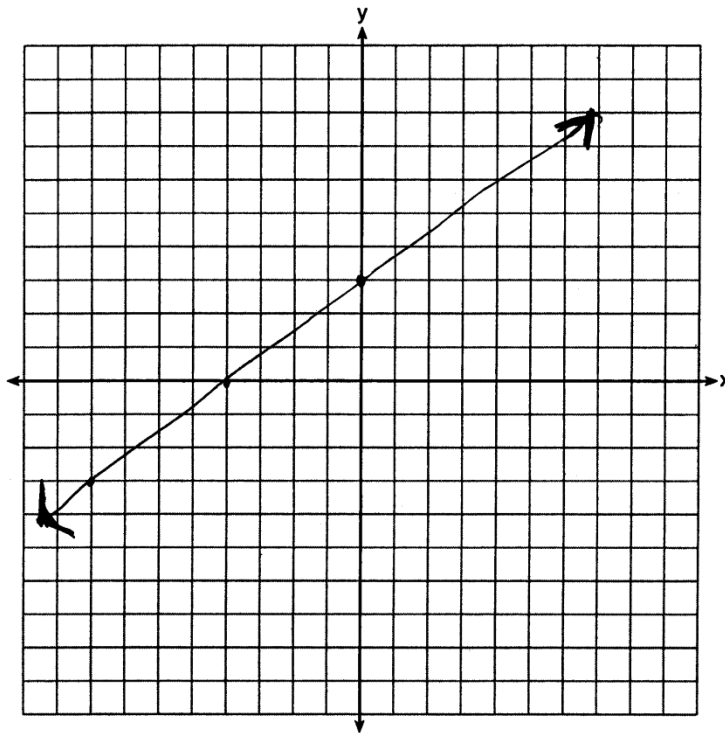
↗ $2 \stackrel{?}{=} \frac{9}{12}$
NO.

The point is
not a solution.

Score 0: The student drew an incorrect graph and did not give an explanation based on the graph.

Question 29

29 On the set of axes below, draw the graph of the equation $y = -\frac{3}{4}x + 3$.



Is the point (3,2) a solution to the equation? Explain your answer based on the graph drawn.

It could be if you plug
it in and it works.

Score 0: The student's graph was incorrect, "no" is not stated, and the explanation is incomplete.

#30

30 The function f has a domain of $\{1, 3, 5, 7\}$ and a range of $\{2, 4, 6\}$.

Could f be represented by $\{(1,2), (3,4), (5,6), (7,2)\}$?

Justify your answer.

Measured CCLS Cluster: F-IF.A

Commentary: The question measures the knowledge and skills described by the standards within F-IF.A because the student is required to understand that a function from one set, the domain, to another set, the range, assigns to each element of the domain exactly one element of the range. Additionally, the question requires the student to employ MP.3, as the student must provide evidence to support his or her claim about the solution.

Rationale: This question asks the student to determine whether a function could be presented by four given ordered pairs given the domain and range of the function. “Domain” refers to the set of input values, while “range” refers to the set of corresponding output values. Additionally, the student must determine whether exactly one output is assigned to each input. As indicated in the rubric, a correct response will state “yes”, with a correct justification given supporting the student’s reasoning. The justification can be presented in either written form or mathematical form which could include creating a graph of the function. The determining factor in demonstrating a thorough understanding is using mathematically sound justifications for the response.

Compare with questions #17, #20, and #21 which also assess F-IF.A.

Sample student responses and scores appear on the following pages.

Question 30

30 The function f has a domain of $\{1, 3, 5, 7\}$ and a range of $\{2, 4, 6\}$.

Could f be represented by $\{(1,2), (3,4), (5,6), (7,2)\}$? *Yes*

Justify your answer.

Yes It can because in a function all numbers in the domain must lead to a self-specific number in the range, meaning one number in the domain cannot have two different numbers in the range.

Score 2: The student has a complete and correct response.

Question 30

30 The function f has a domain of $\{1, 3, 5, 7\}$ and a range of $\{2, 4, 6\}$.

Could f be represented by $\{(1,2), (3,4), (5,6), (7,2)\}$?

Justify your answer.

$f(x) = \{(1,2), (3,4), (5,6), (7,2)\}$ isn't correct
because 2 is repeated in the y twice. This
means that it's not a function.

Score 1: The student made one conceptual error by misinterpreting the definition of a function.

Question 30

30 The function f has a domain of $\{1, 3, 5, 7\}$ and a range of $\{2, 4, 6\}$.

Could f be represented by $\{(1,2), (3,4), (5,6), (7,2)\}$?

Justify your answer.

yes.

Score 0: The student wrote “yes” but a complete justification was not provided.

#31

31 Factor the expression $x^4 + 6x^2 - 7$ completely.

Measured CCLS Cluster: A-SSE.A

Commentary: The question measures the knowledge and skills described by the standards within A-SSE.A because it requires the student to use the structure of an expression to identify ways to rewrite it. Additionally, the question requires the student to employ MP.7 (Look for and make use of structure), as they must recognize the given expression as displaying the same structure as a quadratic expression.

Rationale: This question requires the student to factor the polynomial expression $x^4 + 6x^2 - 7$. As indicated in the rubric, a correct response will include the correct factored form of $(x^2 + 7)(x + 1)(x - 1)$ or an equivalent form including the three binomials, with correct work shown.

One possible way of determining this is by recognizing that $x^4 = (x^2)^2$:

$$x^4 + 6x^2 - 7$$

$$(x^2)^2 + 6(x^2) - 7$$

The expression can then be viewed as a quadratic expression in x^2 , with factors $(x^2 + 7)$ and $(x^2 - 1)$:

$$(x^2 + 7)(x^2 - 1)$$

Recognizing $(x^2 - 1)$ as a difference of squares with factors $(x - 1)$ and $(x + 1)$ leads to the completely factored form:

$$(x^2 + 7)(x - 1)(x + 1)$$

Sample student responses and scores appear on the following pages.

Question 31

31 Factor the expression $x^4 + 6x^2 - 7$ completely.

$$(x^2+7) \quad (x^2-1)$$

$$(x^2+7)(x-1)(x+1)$$

Score 2: The student has a complete and correct response.

Question 31

31 Factor the expression $x^4 + 6x^2 - 7$ completely.

$$x^2(x^2 + 6x - 7)$$
$$x^2(x-1)(x+7)$$

Score 1: The student made an error by factoring out x^2 incorrectly.

Question 31

31 Factor the expression $x^4 + 6x^2 - 7$ completely.

$$(x^2 + 7)(x^2 - 1)$$
$$(x - 1)(x + 1)$$

Score 1: The student made one error by not including the factor $(x^2 + 7)$ in the final answer.

Question 31

31 Factor the expression $x^4 + 6x^2 - 7$ completely.

$$x^4 + 6x^2 - 7$$
$$x^2(x^2 - 6)(x + 7)$$

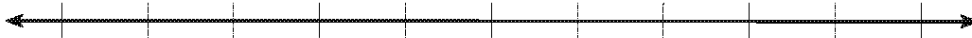
Score 0: The student's response is completely incorrect.

#32

32 Robin collected data on the number of hours she watched television on Sunday through Thursday nights for a period of 3 weeks. The data are shown in the table below.

	Sun	Mon	Tues	Wed	Thurs
Week 1	4	3	3.5	2	2
Week 2	4.5	5	2.5	3	1.5
Week 3	4	3	1	1.5	2.5

Using an appropriate scale on the number line below, construct a box plot for the 15 values.



Measured CCLS Cluster: S-ID.A

Commentary: The question measures the knowledge and skills described by the standards within S-ID.A because it requires the student to represent data with a box plot on the real number line. Additionally, the question encourages students to use tools (in this case, a graphing calculator) strategically, as described by MP.5 (Use appropriate tools strategically).

Rationale: This question requires the student to create a box plot based on the number of hours of television that are watched during a 3 week period. A box plot is a statistical plot that summarizes data by showing five important values: the minimum (“*Min*”), the first quartile (“ Q_1 ”), the median or second quartile (“ Q_2 ”), the third quartile (“ Q_3 ”) and the maximum value (“*Max*”). The student must determine all five values, possibly with the aid of a graphing calculator. As indicated in the rubric, a correct response includes a correct plot with $Min = 1$, $Q_1 = 2$, $Q_2 = 3$, $Q_3 = 4$, and $Max = 5$. The determining factor in demonstrating a thorough understanding is using mathematically sound procedures to lead to a correct box plot, which includes all correct values for *Min*, Q_1 , Q_2 , Q_3 , and *Max*.

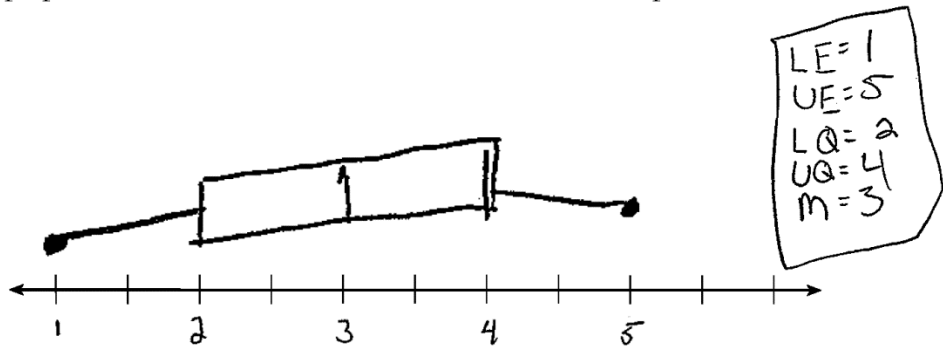
Sample student responses and scores appear on the following pages.

Question 32

32 Robin collected data on the number of hours she watched television on Sunday through Thursday nights for a period of 3 weeks. The data are shown in the table below.

	Sun	Mon	Tues	Wed	Thurs
Week 1	4	3	3.5	2	2
Week 2	4.5	5	2.5	3	1.5
Week 3	4	3	1	1.5	2.5

Using an appropriate scale on the number line below, construct a box plot for the 15 values.



① 1.5, 1.5, ② 2, 2.5, 2.5, 3, 3, 3.5, ④ 4, 4.5, ⑤ 5

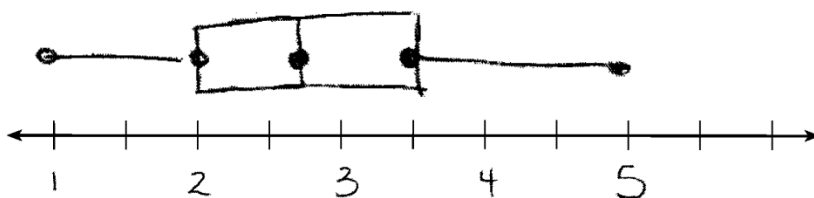
Score 2: The student has a complete and correct response.

Question 32

32 Robin collected data on the number of hours she watched television on Sunday through Thursday nights for a period of 3 weeks. The data are shown in the table below.

	Sun	Mon	Tues	Wed	Thurs
Week 1	4	3	3.5	2	2
Week 2	4.5	5	2.5	3	1.5
Week 3	4	3	1	1.5	2.5

Using an appropriate scale on the number line below, construct a box plot for the 15 values.



min 1
 $Q_1 = 2$
med = 2.75
 $Q_3 = 3.5$
max = 5

Score 1: The student drew an appropriate plot based on an incorrect data set.

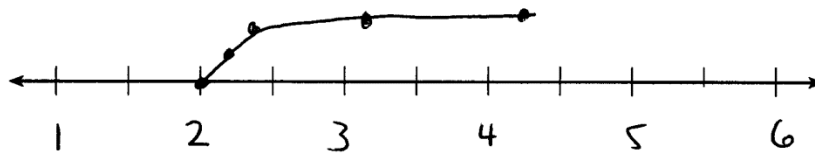
Question 32

32 Robin collected data on the number of hours she watched television on Sunday through Thursday nights for a period of 3 weeks. The data are shown in the table below.

	Sun	Mon	Tues	Wed	Thurs
Week 1	4	3	3.5	2	2
Week 2	4.5	5	2.5	3	1.5
Week 3	4	3	1	1.5	2.5

2.5 11 7 6.5 6

Using an appropriate scale on the number line below, construct a box plot for the 15 values.



$$\frac{12.5}{3} = 4.1\bar{6}$$

$$\frac{7}{3} = 2.\bar{3}$$

$$\frac{6}{3} = 2$$

$$\frac{11}{3} = 3.\bar{6}$$

$$\frac{6.5}{3} = 2.1\bar{6}$$

Score 0: The student's response is completely irrelevant.

#33

33 Write an equation that defines $m(x)$ as a trinomial where $m(x) = (3x - 1)(3 - x) + 4x^2 + 19$.

Solve for x when $m(x) = 0$.

Measured CCLS Cluster: A-REI.B

Commentary: The question measures the knowledge and skills described by the standards within A-REI.B because it requires the student to solve a quadratic equation using a method of their choosing which could include completing the square, the quadratic formula or factoring. Secondly, the question measures knowledge and skills described by the standard within A-APR.A because the student is required to perform arithmetic operations on a given polynomial.

Rationale: This question requires students to perform arithmetic operations on a polynomial, then to solve the resulting quadratic equation. As indicated in the rubric, a correct response will include $m(x) = x^2 + 10x + 16$ or an equivalent trinomial equation and -8 and -2 and correct work shown. The determining factor in demonstrating a thorough understanding is using mathematically sound procedures to lead to a correct response.

The correct answer may be arrived at by applying properties of operations to rewrite the polynomial using three terms (as a trinomial), then solving the quadratic equation found when rewriting the polynomial, by factoring.

Writing $m(x)$ as a trinomial:

$$m(x) = (3x - 1)(3 - x) + 4x^2 + 19$$

$$m(x) = (9x - 3 - 3x^2 + x) + 4x^2 + 19$$

$$m(x) = (10x - 3 - 3x^2) + 4x^2 + 19$$

$$m(x) = x^2 + 10x + 16$$

Solving for x when $m(x) = 0$:

$$x^2 + 10x + 16 = 0$$

$$(x + 8)(x + 2) = 0$$

$$x = -8 \text{ and } x = -2$$

Compare with questions #8 and #27, which also assess A-REI.B.

Sample student responses and scores appear on the following pages.

Question 33

33 Write an equation that defines $m(x)$ as a trinomial where $m(x) = (3x - 1)(3 - x) + 4x^2 + 19$.

$$(3x-1)(3-x) + (4x^2 + 19) = m(x)$$

$$9x - 3x^2 - 3 + 1x + 4x^2 + 19 = m(x)$$

$$10x + x^2 + \frac{16}{\cancel{2}} = m(x)$$

Solve for x when $m(x) = 0$.

$$m(x) = 10x + x^2 + \frac{16}{\cancel{2}} \quad \text{---} \quad x^2 + 10x + \frac{16}{\cancel{2}}$$

$$0 = (x+8)(x+2)$$

$$x+8=0 \quad \text{or} \quad x+2=0$$
$$\begin{matrix} -8 & -8 & & -2 & -2 \end{matrix}$$

$$x = -8 \quad x = -2$$

Score 4: The student has a complete and correct response.

Question 33

33 Write an equation that defines $m(x)$ as a trinomial where $m(x) = (3x - 1)(3 - x) + 4x^2 + 19$.

$$\begin{array}{r|l} 3x & -1 \\ \hline 3 & 9x & -3 \\ -x & -3x^2 & -1x \end{array}$$

$$\begin{aligned} & (3x-1)(3-x) + 4x^2 + 19 \\ & (-3x^2 + 10x - 3) + (4x^2 + 19) \\ & x^2 + 10x + 16 \end{aligned}$$

Solve for x when $m(x) = 0$.

$$\begin{aligned} x^2 + 10x + 16 &= 0 \\ (x+8)(x+2) & \end{aligned}$$

$$\begin{array}{r} x+8=0 \\ -8 \quad -8 \\ \hline x = -8 \end{array}$$

$$\begin{array}{r} x+2=0 \\ -2 \quad -2 \\ \hline x = -2 \end{array}$$

$$x = -8, -2$$

Score 3: The student wrote an appropriate expression instead of an equation.

Question 33

33 Write an equation that defines $m(x)$ as a trinomial where $m(x) = (3x - 1)(3 - x) + 4x^2 + 19$.

$$(3x-1)(3-x) + 4x^2 + 19$$
$$\boxed{9x} \quad \boxed{-3x^2} \quad \boxed{-3} \quad \boxed{+3x} \quad \boxed{+4x^2} \quad \boxed{+19}$$
$$x^2 + 8x + 16$$

Solve for x when $m(x) = 0$.

$$0 = x^2 + 8x + 16$$
$$(x+4)(x+4)$$
$$x+4 = 0 \quad x+4 = 0$$
$$x = -4 \quad x = -4$$

Score 2: The student wrote an incorrect trinomial that was not set to $m(x)$, but solved it appropriately.

Question 33

33 Write an equation that defines $m(x)$ as a trinomial where $m(x) = (3x - 1)(3 - x) + 4x^2 + 19$.

3 3 3 3 3 3 3 3

$$3x - 1 \cdot 3 - x + 4x^2 + 19$$

$$(3x - 1)(3 - x)$$

$$m(x) = x^2 + 10x + 19$$

$$9x - 3x^2 - 3 + x$$

$$10x - 3x^2 - 3 + 4x^2 + 19$$

Solve for x when $m(x) = 0$.

$$m(x) = x^2 + 10x + 19$$

$$m(x) = (x \quad)(x \quad)$$

Score 1: The student showed appropriate work to find $m(x)$, but made one computational error. No further correct work is shown.

Question 33

33 Write an equation that defines $m(x)$ as a trinomial where $m(x) = (3x - 1)(3 - x) + 4x^2 + 19$.

$$3x - 1 + 3 - x + 4x^2 + 19$$

$$3x - 2 - 2 + 4x^2 + 19$$

$$3x - 4 + 4x^2 + 19$$

$$\begin{array}{r} -19 \qquad +19 \\ \hline \end{array}$$

$$3x + 15 + 4x^2$$

Solve for x when $m(x) = 0$.

$$3(0) - 15 + 4x^2$$

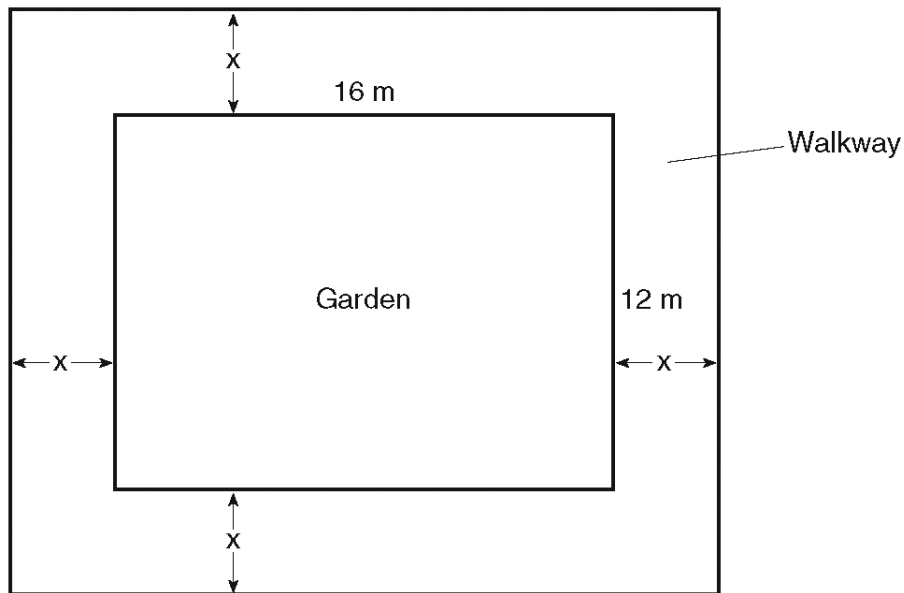
$$0 - 15 + 4x^2$$

$$\begin{array}{r} -15 \quad -15 \\ \hline \end{array}$$

$$0 = 1$$

Score 0: The student's response is completely incorrect.

- 34 A rectangular garden measuring 12 meters by 16 meters is to have a walkway installed around it with a width of x meters, as shown in the diagram below. Together, the walkway and the garden have an area of 396 square meters.



Write an equation that can be used to find x , the width of the walkway.

Describe how your equation models the situation.

Determine and state the width of the walkway, in meters.

Measured CCLS Cluster: A-CED.A

Commentary: The question measures the knowledge and skills described by standards within A-CED.A because it requires the student to create an equation in one variable and use the equation to solve a problem. In contrast to questions that simply ask the student to solve an equation, this question also requires the student to employ MP.4 in creating a model (an equation) representing a situation. Additionally, the student must employ MP.2 in explaining how an abstract model relates to the context described by the problem.

Rationale: The question asks the student to write an equation that can be used to find the width of the walkway, to describe how their equation models the situation, and to apply the equation created in order to determine the width of the walkway. As indicated in the rubric, a correct response will include the equation $(2x + 12)(2x + 16) = 396$ or an equivalent equation, a correct description is given, and correct work is shown to find 3. The determining factor in demonstrating a thorough understanding is using mathematically sound procedures to create a viable model, a correct description of the model and a correct solution where the student applies the model created.

The student arrived at the correct answer by writing the equation $(2x + 16)(2x + 12) = 396$.

A correct description of the equation as it relates to the situation is written, including ideas such as:

- $(2x + 16)$ represents one dimension of the combined garden and walkway.
- $(2x + 12)$ represents the other dimension of the combined garden and walkway.
- The area of the combined garden and walkway is the product of $(2x + 16)$ and $(2x + 12)$, and is equal to 396 square meters.

Lastly, the student correctly solved the quadratic equation created in order to determine the length of the walkway. The student has the choice of how to solve the quadratic equation since the method to solve is not specified in the question.

$$(2x + 16)(2x + 12) = 396$$

$$4x^2 + 32x + 24x + 192 = 396$$

$$4x^2 + 56x - 204 = 0$$

$$4(x^2 + 14x - 51) = 0$$

$$x^2 + 14x - 51 = 0$$

$$(x + 17)(x - 3) = 0$$

$$x = -17 \text{ and } x = 3$$

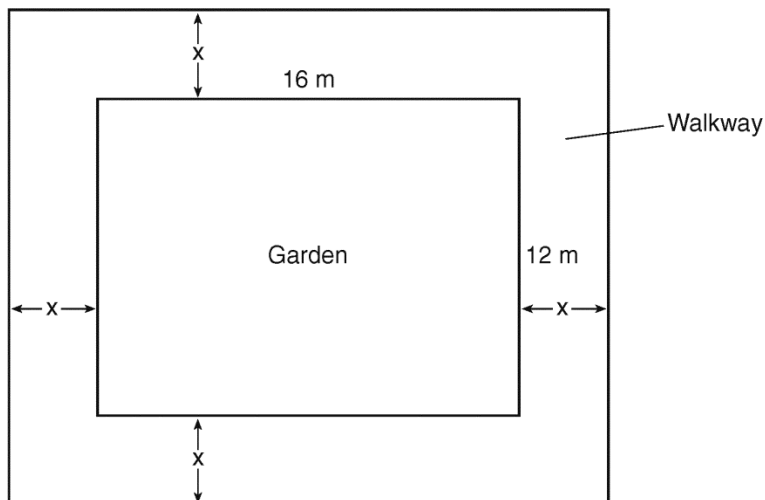
Since 3 is the positive solution of the quadratic equation, the walkway is 3 meters wide. While -17 is a solution to the equation $(2x + 16)(2x + 12) = 396$, it is not appropriate within this real-world modeling context.

Compare with question #36, which also assesses A-CED.A.

Sample student responses and scores appear on the following pages.

Question 34

34 A rectangular garden measuring 12 meters by 16 meters is to have a walkway installed around it with a width of x meters, as shown in the diagram below. Together, the walkway and the garden have an area of 396 square meters.



Write an equation that can be used to find x , the width of the walkway.

$$(2x + 16)(2x + 12) = 396$$

Describe how your equation models the situation.

It is the length plus the walkway \times width plus the walkway

Determine and state the width of the walkway, in meters.

$$(2x + 16)(2x + 12) = 396$$

$$4x^2 + 32x + 24x + 192 = 396$$

$$4x^2 + 56x - 204 = 0$$

$$x^2 + 14x - 51 = 0$$

$$(x + 17)(x - 3) = 0$$

$$x + 17 = 0 \quad x - 3 = 0$$

$$x = -17, 3$$

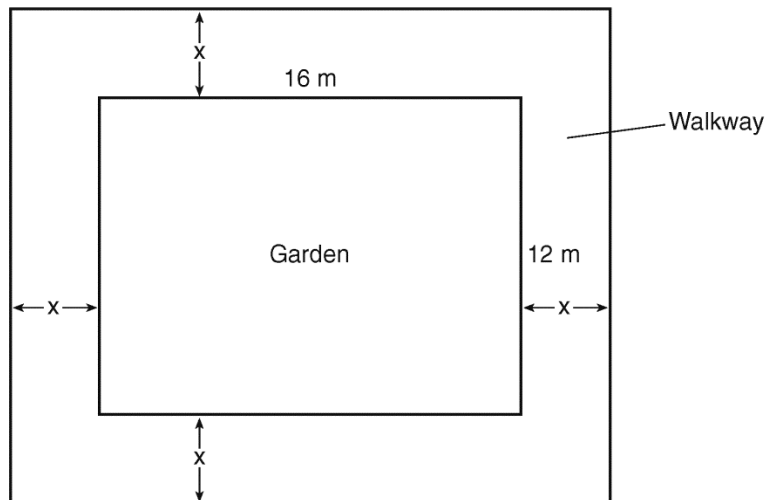
3 meters

The width of the walkway is 3 meters

Score 4: The student has a complete and correct response.

Question 34

34 A rectangular garden measuring 12 meters by 16 meters is to have a walkway installed around it with a width of x meters, as shown in the diagram below. Together, the walkway and the garden have an area of 396 square meters.



Write an equation that can be used to find x , the width of the walkway.

$$(2x+16)(2x+12) = 396$$

Describe how your equation models the situation.

The area is the length \times width including the walkway around the garden.

Determine and state the width of the walkway, in meters.

$$\begin{aligned} 4x^2 + 24x + 32x + 192 &= 396 \\ 4x^2 + 56x + 192 &= 396 \\ 4x^2 + 56x - 204 &= 0 \\ x^2 + 14x - 51 &= 0 \end{aligned}$$

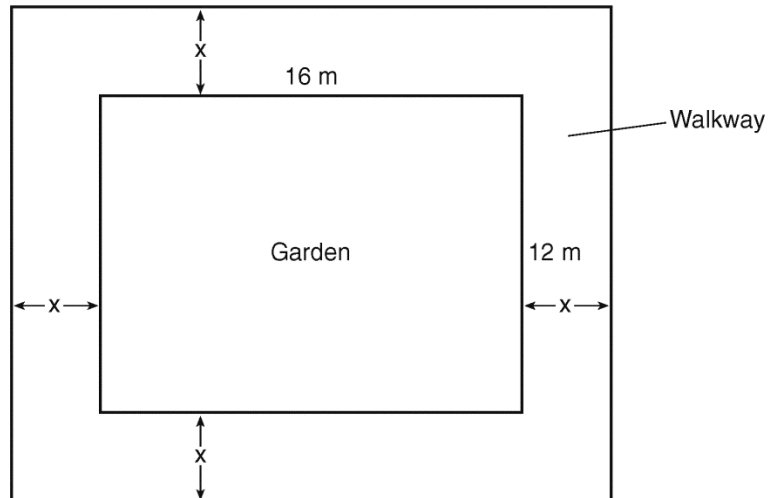
$$(x-17)(x+3) = 0$$

$x = 17$ ~~$x = -3$~~

Score 3: The student wrote the correct equation and an appropriate description, but made one factoring error.

Question 34

34 A rectangular garden measuring 12 meters by 16 meters is to have a walkway installed around it with a width of x meters, as shown in the diagram below. Together, the walkway and the garden have an area of 396 square meters.



Write an equation that can be used to find x , the width of the walkway.

$$(2x+16)(2x+12) = 396$$

Describe how your equation models the situation.

It shows the total area of the walkway and the garden.

Determine and state the width of the walkway, in meters.

$$4x^2 + 24x + 32x + 192 = 396$$

$$4x^2 + 56x + 192 = 396$$

$$\underline{-396 \quad -396}$$

$$4x^2 + 56x - 204 = 0$$

$$x^2 + 14x - 204 = 0$$

$$(x-17)(x+12) = 0$$

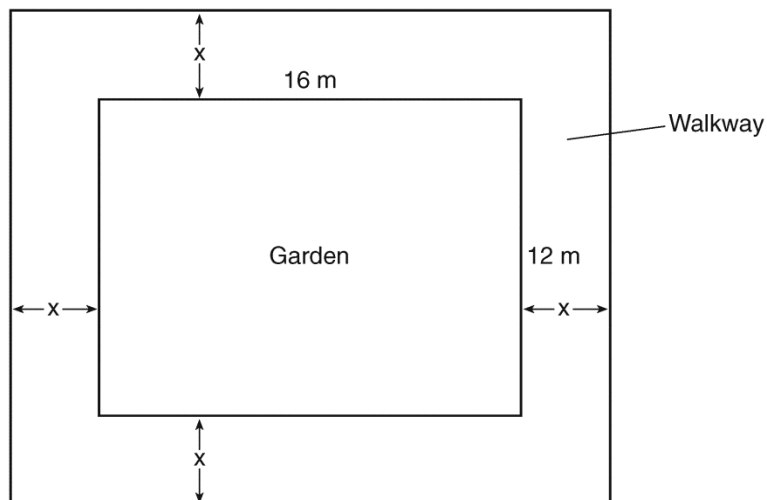
$$x = 17 \quad x = -12$$

$$x = 17 \text{ meters}$$

Score 2: The student made one computational error when dividing by 4 and one factoring error.

Question 34

34 A rectangular garden measuring 12 meters by 16 meters is to have a walkway installed around it with a width of x meters, as shown in the diagram below. Together, the walkway and the garden have an area of 396 square meters.



Write an equation that can be used to find x , the width of the walkway.

$$(16 + x + x)(12 + x + x) = 396$$

Describe how your equation models the situation.

The length times the width of a rectangle gives you the area.

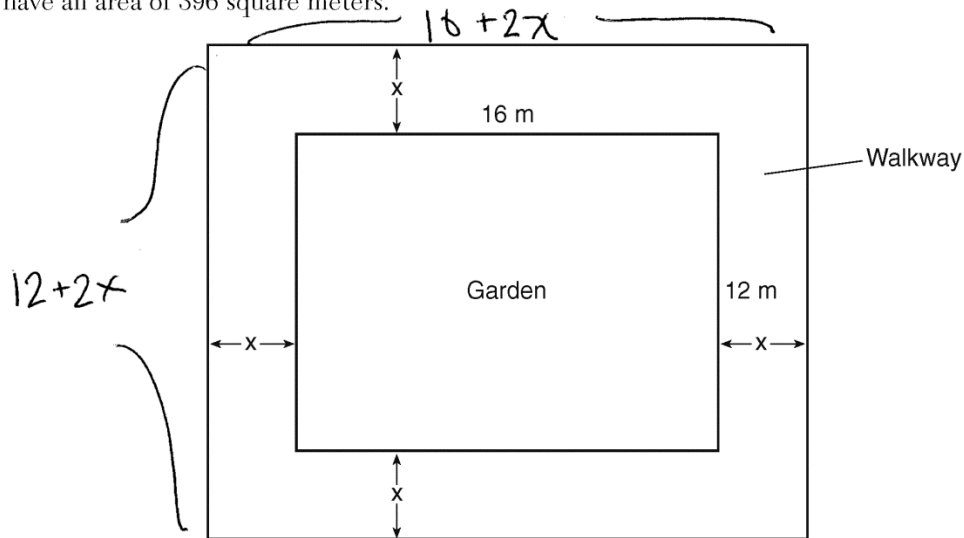
Determine and state the width of the walkway, in meters.

$$\begin{aligned}(16 + 2x)(12 + 2x) &= 396 \\ 16 + 24x + 2x &= 396 \\ 16 + 26x &= 396 \\ 26x &= 380 \\ x &= 14.62 \text{ m}\end{aligned}$$

Score 1: The student made one conceptual error in solving the equation and gave an incomplete description by not including the walkway.

Question 34

34 A rectangular garden measuring 12 meters by 16 meters is to have a walkway installed around it with a width of x meters, as shown in the diagram below. Together, the walkway and the garden have an area of 396 square meters.



Write an equation that can be used to find x , the width of the walkway.

$$(12 + 2x) \times (16 + 2x) = 284$$

Describe how your equation models the situation.

Length \times Width

Determine and state the width of the walkway, in meters.

$$192 + 24x + 32x + 4x = 284$$

$$192 + 60x = 284$$

$$60x = 92$$

$$x = 1.5\bar{3}$$

Score 0: The student's response is completely incorrect.

#35

- 35 Caitlin has a movie rental card worth \$175. After she rents the first movie, the card's value is \$172.25. After she rents the second movie, its value is \$169.50. After she rents the third movie, the card is worth \$166.75.

Assuming the pattern continues, write an equation to define $A(n)$, the amount of money on the rental card after n rentals.

Caitlin rents a movie every Friday night. How many weeks in a row can she afford to rent a movie, using her rental card only? Explain how you arrived at your answer.

Measured CCLS Cluster: F-BF.A

Commentary: The question measures knowledge and skills described by standards within F-BF.A because it requires the student to determine an explicit expression given a real-world context. This question requires the student to employ MP.4 in creating a model, as well as MP.3 in justifying their solution based on the model.

Rationale: This question requires the student to create a model representing a given situation and to apply the model in order to determine how many weeks a video can be rented. As indicated in the rubric, a correct response includes the equation $A(n) = 175 - 2.75n$, correct work to find 63, and a correct explanation describing how the student was able to determine the number of weeks. The determining factor in demonstrating a thorough understanding is using mathematically sound procedures to lead to a correct response.

The correct model can be found using multiple strategies, one of which is:

Week 0 = \$175, Week 1 = \$172.25

$$m = \frac{(175 - 172.25)}{(1 - 0)} = 2.75$$

Therefore, the equation representing the amount of money on the rental card after n rentals is $A(n) = 175 - 2.75n$ since 2.75 represents the amount by which the rental card value decreases each week.

$$0 = 175 - 2.75n$$

$$2.75n = 175$$

$$n = 63.6363 \dots$$

Therefore, Caitlin can rent a movie each week for 63 weeks since she will not have enough money on the rental card to rent a movie for the 64th week.

Sample student responses and scores appear on the following pages.

Question 35

35 Caitlin has a movie rental card worth \$175. After she rents the first movie, the card's value is \$172.25. After she rents the second movie, its value is \$169.50. After she rents the third movie, the card is worth \$166.75.

Assuming the pattern continues, write an equation to define $A(n)$, the amount of money on the rental card after n rentals.

$$A(n) = 175 - 2.75n$$

Caitlin rents a movie every Friday night. How many weeks in a row can she afford to rent a movie, using her rental card only? Explain how you arrived at your answer.

$$0 = 175 - 2.75n$$

$$2.75n = 175$$

$$n = 63.\overline{6363}$$

She can watch for 63 weeks because at the 64th week she won't have enough money to rent a movie.

Score 4: The student has a complete and correct response.

Question 35

35 Caitlin has a movie rental card worth \$175. After she rents the first movie, the card's value is \$172.25. After she rents the second movie, its value is \$169.50. After she rents the third movie, the card is worth \$166.75.

Assuming the pattern continues, write an equation to define $A(n)$, the amount of money on the rental card after n rentals.

$$A(n) = 175 - 2.75n$$

Caitlin rents a movie every Friday night. How many weeks in a row can she afford to rent a movie, using her rental card only? Explain how you arrived at your answer.

$$0 = 175 - 2.75n$$

$$-175 = -2.75n$$

$$63.\overline{63} = n$$

She can watch movies for 64 weeks. I found my answer because I rounded the number of weeks that I found.

Score 3: The student made an error by stating the incorrect number of weeks.

Question 35

35 Caitlin has a movie rental card worth \$175. After she rents the first movie, the card's value is \$172.25. After she rents the second movie, its value is \$169.50. After she rents the third movie, the card is worth \$166.75.

Assuming the pattern continues, write an equation to define $A(n)$, the amount of money on the rental card after n rentals.

$$175 - 2.75n$$

Caitlin rents a movie every Friday night. How many weeks in a row can she afford to rent a movie, using her rental card only? Explain how you arrived at your answer.

$$0 = 175 - 2.75n$$

$$2.75n = 175$$

$$n = 63.\overline{63}$$

64 weeks because I rounded up

Score 2: The student wrote an appropriate expression instead of an equation and made one rounding error.

Question 35

35 Caitlin has a movie rental card worth \$175. After she rents the first movie, the card's value is \$172.25. After she rents the second movie, its value is \$169.50. After she rents the third movie, the card is worth \$166.75.

Assuming the pattern continues, write an equation to define $A(n)$, the amount of money on the rental card after n rentals.

$$\begin{array}{cccc} 175, & 172.25, & 169.50, & 166.75 \\ & \downarrow & \downarrow & \downarrow \\ & 2.75 & 2.75 & 2.75 \end{array}$$
$$a_n = a_1 + (n-1)d$$
$$a_n = 175 + (n-1)2.75$$

Caitlin rents a movie every Friday night. How many weeks in a row can she afford to rent a movie, using her rental card only? Explain how you arrived at your answer.

$$0 = 175 + 2.75n - 2.75$$
$$5.5n = 175$$
$$n = 31.8$$

32 weeks because
that is when the card
will not have any money.

Score 1: The student made an error in the equation and one computational error in solving the equation. The student stated an incorrect number of weeks.

Question 35

35 Caitlin has a movie rental card worth \$175. After she rents the first movie, the card's value is \$172.25. After she rents the second movie, its value is \$169.50. After she rents the third movie, the card is worth \$166.75.

Assuming the pattern continues, write an equation to define $A(n)$, the amount of money on the rental card after n rentals.

$$175 - n$$

Caitlin rents a movie every Friday night. How many weeks in a row can she afford to rent a movie, using her rental card only? Explain how you arrived at your answer.

$$175 - 2.25 = 172.25$$

$$2.25$$

$$\times 10$$

$$\hline 22.5$$

$$2.25$$

$$\times 78$$

$$\hline 175.5$$

78 weeks until the card runs out

Score 0: The student's response is completely incorrect.

#36

36 An animal shelter spends \$2.35 per day to care for each cat and \$5.50 per day to care for each dog. Pat noticed that the shelter spent \$89.50 caring for cats and dogs on Wednesday.

Write an equation to represent the possible numbers of cats and dogs that could have been at the shelter on Wednesday.

Pat said that there might have been 8 cats and 14 dogs at the shelter on Wednesday. Are Pat's numbers possible? Use your equation to justify your answer.

Later, Pat found a record showing that there were a total of 22 cats and dogs at the shelter on Wednesday. How many cats were at the shelter on Wednesday?

Measured CCLS Cluster: A-CED.A

Commentary: The question measures the knowledge and skills described by the standards within A-CED.A because the student is required to represent constraints using a system of equations. This question requires the student to employ MP.4, as the student is required to create a model, and MP.2, as the student is required to reason quantitatively using the model. Finally, the question also signals a shift in high school-level modeling as the student must create an equation with variables of his or her own choosing, rather than use variables defined in the question prompt.

Rationale: This question requires the student to write an equation representing a given situation, determine whether a given scenario is a viable situation based on the equation created, and to apply the model or equation created to determine the number of cats. As indicated in the rubric, a correct response will include $2.35c + 5.50d = 89.50$ or an equivalent equation, “no,” and a correct justification, and correct work to find 10. The determining factor in demonstrating a thorough understanding is creating a valid model to represent the constraints and utilizing the model to show a thorough understanding of the context.

A correct equation representing the total possible number of cats and dogs at the shelter is:

$$2.35c + 5.50d = 89.50$$

In order to determine whether it is possible that 8 cats and 14 dogs could have been at the shelter on Wednesday, the student could substitute into the model created to determine that it is not a possible combination:

$$2.35(8) + 5.50(14) = 89.50$$

$$18.80 + 77 = 89.50$$

$$95.80 \neq 89.50$$

Therefore, it isn't possible that there were 8 cats and 14 dogs at the shelter on Wednesday.

In order to determine the number of cats at the shelter on Wednesday when there was a total of 22 cats and dogs, the student can create a new equation representing the total number of animals: $c + d = 22$, then use that equation to solve a system of equations, as shown below.

$$2.35c + 5.50d = 89.50$$

$$c + d = 22$$

$$\text{Then } c = 22 - d.$$

$$\text{Therefore, } 2.35(22 - d) + 5.50d = 89.50$$

$$51.7 - 2.35d + 5.50d = 89.50$$

$$3.15d = 37.8$$

$$d = 12$$

$$\text{Since } c + d = 22$$

$$c + 12 = 22$$

$$\text{Therefore, } c = 10$$

Compare with question #36, which also assesses A-CED.A.

Sample student responses and scores appear on the following pages.

Question 36

36 An animal shelter spends \$2.35 per day to care for each cat and \$5.50 per day to care for each dog. Pat noticed that the shelter spent \$89.50 caring for cats and dogs on Wednesday.

Write an equation to represent the possible numbers of cats and dogs that could have been at the shelter on Wednesday.

$$2.35c + 5.50d = 89.50$$

Pat said that there might have been 8 cats and 14 dogs at the shelter on Wednesday. Are Pat's numbers possible? Use your equation to justify your answer.

$$2.35 \cdot 8 = 18.8$$

$$5.50 \cdot 14 = 77$$

$$18.8 + 77 = 95.8 \text{ so it isn't true}$$

Later, Pat found a record showing that there were a total of 22 cats and dogs at the shelter on Wednesday. How many cats were at the shelter on Wednesday?

$$\begin{array}{r} \cancel{2.35}c + 5.50d = 89.50 \\ -2.35(c + d = 22) \\ \hline \cancel{-2.35}c \cancel{-} -2.35d = -51.70 \\ \hline 3.15d = 37.8 \\ d = 12 \end{array}$$
$$\begin{array}{l} c + d = 22 \\ c + 12 = 22 \\ c = 10 \\ 10 \rightarrow \text{cats} \end{array}$$

Score 4: The student has a complete and correct response.

Question 36

36 An animal shelter spends \$2.35 per day to care for each cat and \$5.50 per day to care for each dog. Pat noticed that the shelter spent \$89.50 caring for cats and dogs on Wednesday.

Write an equation to represent the possible numbers of cats and dogs that could have been at the shelter on Wednesday.

$$2.35x + 5.50y = 89.50$$

Pat said that there might have been 8 cats and 14 dogs at the shelter on Wednesday. Are Pat's numbers possible? Use your equation to justify your answer.

$$2.35(8) + 5.50(\cancel{14}) = 89.50$$

$$18.8 + 77 = 89.50 \quad \times$$

8 and 14 can't work.

Later, Pat found a record showing that there were a total of 22 cats and dogs at the shelter on Wednesday. How many cats were at the shelter on Wednesday?

$$2.35x + 5.50y = 89.50 \quad x + y = 22$$

$$y = 22 - x$$

$$2.35x + 5.50(22 - x) = 22$$

$$2.35x + 121 - 5.50x = 22$$

$$-3.15x = -99$$

$$\text{about } 31 \text{ cats} \quad x = 31.4$$

Score 3: The student showed appropriate work, but wrote 22 instead of 89.50.

Question 36

36 An animal shelter spends \$2.35 per day to care for each cat and \$5.50 per day to care for each dog. Pat noticed that the shelter spent \$89.50 caring for cats and dogs on Wednesday.

Write an equation to represent the possible numbers of cats and dogs that could have been at the shelter on Wednesday.

$$2.35x + 5.50y = \$89.50$$

Pat said that there might have been 8 cats and 14 dogs at the shelter on Wednesday. Are Pat's numbers possible? Use your equation to justify your answer.

$$\begin{array}{r} 89.50 \\ - 77 \\ \hline 12.5 \\ - 18.8 \\ \hline -6.3 \end{array}$$

no, Prices can't be negative

Later, Pat found a record showing that there were a total of 22 cats and dogs at the shelter on Wednesday. How many cats were at the shelter on Wednesday?

$$\begin{array}{l} x + y = 22 \\ 2.35x + 5.50y = 89.50 \\ \hline 3.35x + 6.50y = 111.5 \end{array}$$

Score 2: The student showed a correct equation, stated "no" and wrote a correct justification, but no further correct work was shown.

Question 36

36 An animal shelter spends \$2.35 per day to care for each cat and \$5.50 per day to care for each dog. Pat noticed that the shelter spent \$89.50 caring for cats and dogs on Wednesday.

Write an equation to represent the possible numbers of cats and dogs that could have been at the shelter on Wednesday.

Pat said that there might have been 8 cats and 14 dogs at the shelter on Wednesday. Are Pat's numbers possible? Use your equation to justify your answer.

Later, Pat found a record showing that there were a total of 22 cats and dogs at the shelter on Wednesday. How many cats were at the shelter on Wednesday?

$$\begin{array}{r} 2.35 \\ \times 10 \\ \hline 23.50 \end{array}$$

$$\begin{array}{r} 89.50 \\ - 23.50 \\ \hline 66.00 \end{array}$$

$$66 \div 5.50 = 12$$

10 cats

Score 1: The student wrote 10 cats, but appropriate work was not shown.

Question 36

36 An animal shelter spends \$2.35 per day to care for each cat and \$5.50 per day to care for each dog. Pat noticed that the shelter spent \$89.50 caring for cats and dogs on Wednesday.

Write an equation to represent the possible numbers of cats and dogs that could have been at the shelter on Wednesday.

$$\begin{array}{l} 2.35 \\ 5.50 = \$89.50 \end{array}$$

Pat said that there might have been 8 cats and 14 dogs at the shelter on Wednesday. Are Pat's numbers possible? Use your equation to justify your answer.

$$\begin{array}{l} 2.35 \cdot 14 = 32.9 \\ 5.50 \cdot 8 = 44 \\ \text{can't be true} \end{array}$$

Later, Pat found a record showing that there were a total of 22 cats and dogs at the shelter on Wednesday. How many cats were at the shelter on Wednesday?

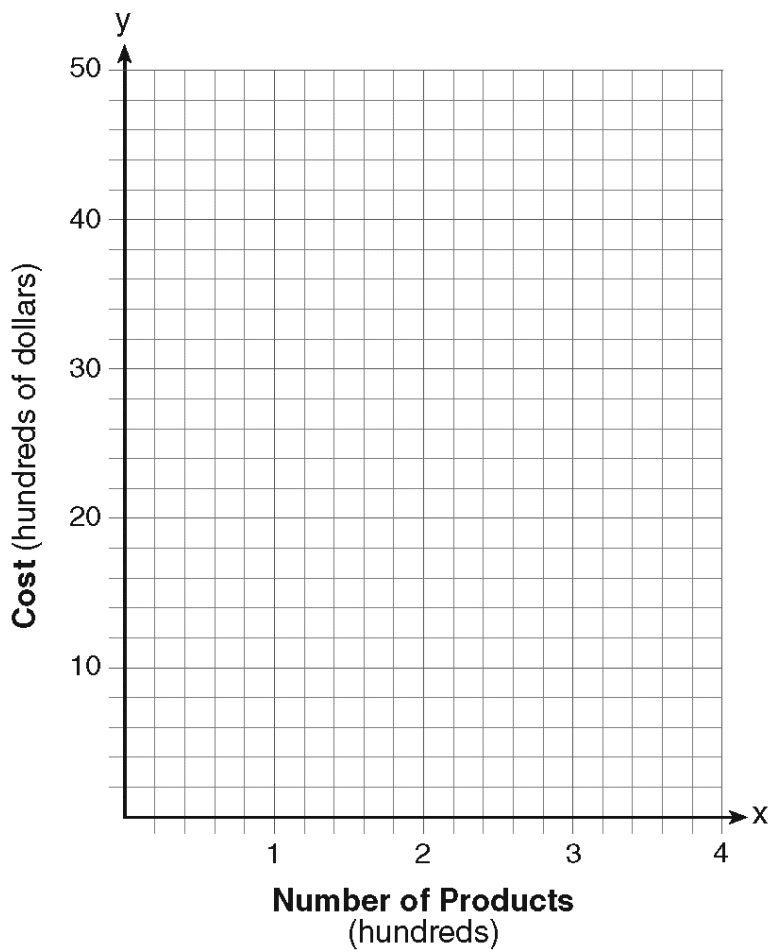
$$\begin{array}{l} 11 \text{ cats} \\ 11 \text{ dogs} \end{array}$$

Score 0: The student's responses are completely incorrect.

#37

37 A company is considering building a manufacturing plant. They determine the weekly production cost at site A to be $A(x) = 3x^2$ while the production cost at site B is $B(x) = 8x + 3$, where x represents the number of products, *in hundreds*, and $A(x)$ and $B(x)$ are the production costs, *in hundreds of dollars*.

Graph the production cost functions on the set of axes below and label them site A and site B .



State the positive value(s) of x for which the production costs at the two sites are equal.
Explain how you determined your answer.

If the company plans on manufacturing 200 products per week, which site should they use?
Justify your answer.

Measured CCLS Cluster: A-REI.D

Commentary: The question measures the knowledge and skills described by standards within A-REI.D because it requires the student to solve the equation $A(x) = B(x)$ using the graphs of the functions A and B . Beyond demonstrating the procedures of graphing and identifying solutions, the student is required to interpret the meaning of the solution in context. The item requires employing a variety of mathematical practices as the student must justify the claim about the solution (MP.3), and interpret the meaning of a mathematical model (MP.4).

Rationale: This question asks the student to graph the given production-cost functions, to determine the positive value of x when the production costs at the sites are equal, and to explain how they determined their answer. The student must graph both functions and use the fact that the graphs intersect in the first quadrant at the point (3,27) as justification for the claim that the production costs at the two sites are equal when $x = 3$ (which corresponds to 300 products). Further interpretation of the functions and their graphs is then required to understand that site A should be used if the company will be manufacturing 200 products per week, since site A 's production cost will be less than site B 's, for 200 products.

As indicated in the rubric, a correct response includes both functions graphed and labeled correctly, 3 and a correct explanation is given, and site A and a correct justification. The determining factor in demonstrating a thorough explanation is providing evidence that the graphs of the function intersect at the point (3,27).

Students need to be aware of the units in the problem and that both axes employ a scale of 100.

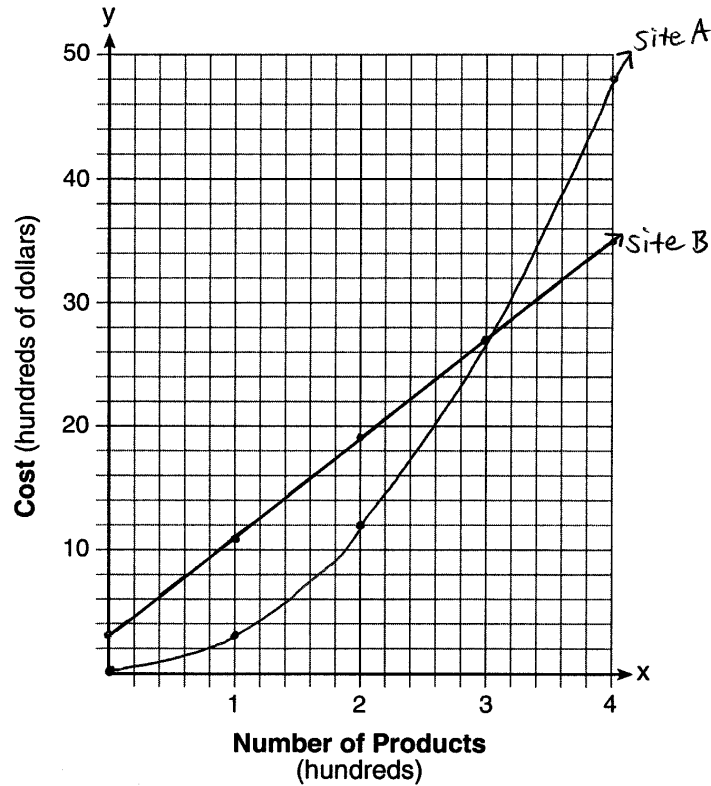
Compare with question #29, which also assesses A-REI.D.

Sample student responses and scores appear on the following pages.

Question 37

37 A company is considering building a manufacturing plant. They determine the weekly production cost at site A to be $A(x) = 3x^2$ while the production cost at site B is $B(x) = 8x + 3$, where x represents the number of products, *in hundreds*, and $A(x)$ and $B(x)$ are the production costs, *in hundreds of dollars*.

Graph the production cost functions on the set of axes below and label them site A and site B.



Question 37 is continued on the next page.

Question 37

Question 37 continued

State the positive value(s) of x for which the production costs at the two sites are equal.
Explain how you determined your answer.

$x=3$ that is where the two graphs
cross each other

If the company plans on manufacturing 200 products per week, which site should they use?
Justify your answer.

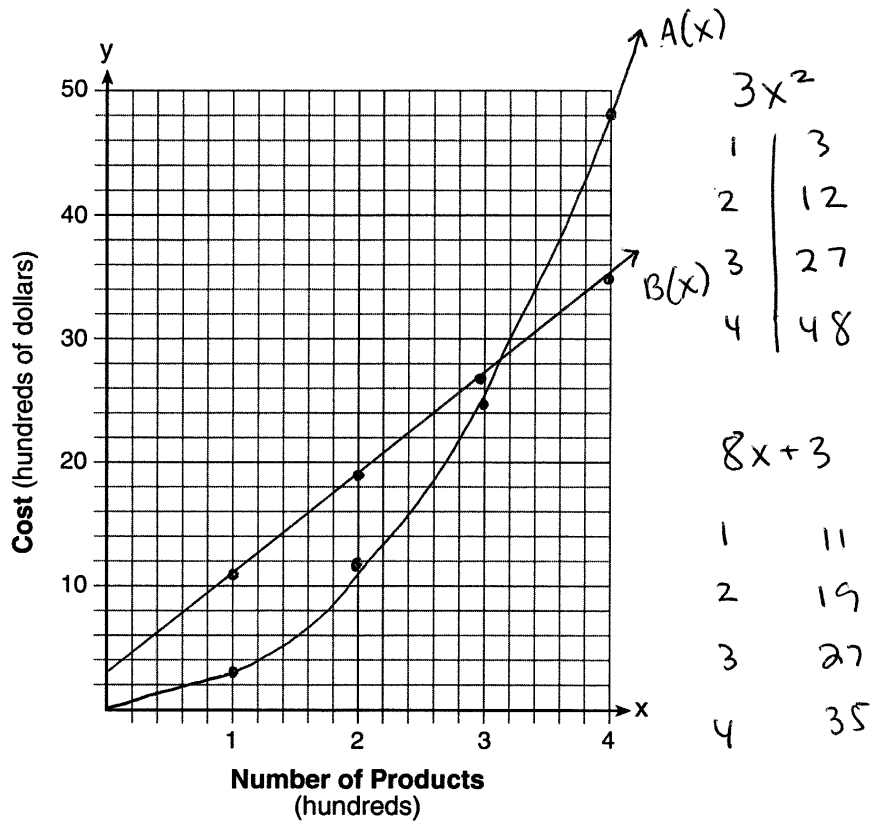
they should use site A because
it costs less. At 200 on the
graph, site A is less than site B.

Score 6: The student has a complete and correct response.

Question 37

37 A company is considering building a manufacturing plant. They determine the weekly production cost at site A to be $A(x) = 3x^2$ while the production cost at site B is $B(x) = 8x + 3$, where x represents the number of products, *in hundreds*, and $A(x)$ and $B(x)$ are the production costs, *in hundreds of dollars*.

Graph the production cost functions on the set of axes below and label them site A and site B.



Question 37 is continued on the next page.

Question 37

Question 37 continued

State the positive value(s) of x for which the production costs at the two sites are equal.
Explain how you determined your answer.

3. The tables meet at $(3, 27)$ so
that's when they are equal.

If the company plans on manufacturing 200 products per week, which site should they use?
Justify your answer.

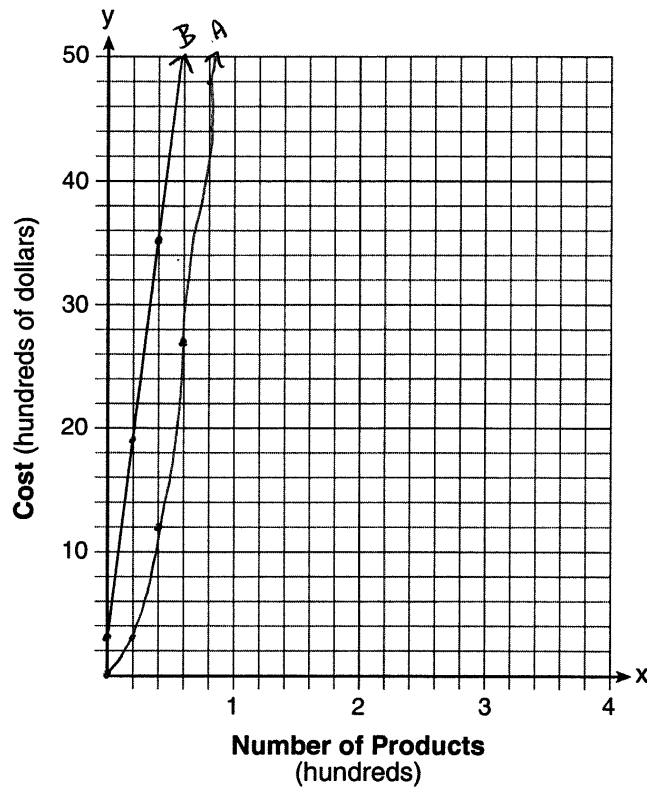
B is higher on the graph at 200, so
A would be cheaper to use.

Score 5: The student made one graphing error at $x = 3$.

Question 37

37 A company is considering building a manufacturing plant. They determine the weekly production cost at site A to be $A(x) = 3x^2$ while the production cost at site B is $B(x) = 8x + 3$, where x represents the number of products, *in hundreds*, and $A(x)$ and $B(x)$ are the production costs, *in hundreds of dollars*.

Graph the production cost functions on the set of axes below and label them site A and site B.



Question 37 is continued on the next page.

Question 37

Question 37 continued

State the positive value(s) of x for which the production costs at the two sites are equal.
Explain how you determined your answer.

Since the graphs don't cross,
the costs aren't equal.

If the company plans on manufacturing 200 products per week, which site should they use?
Justify your answer.

Site A would be cheaper because the
point is less than site B so site A would
have lower costs.

Score 4: The student made a conceptual error when graphing, but had an appropriate response and explanation based on the graph. The student stated site A and gave an appropriate explanation.

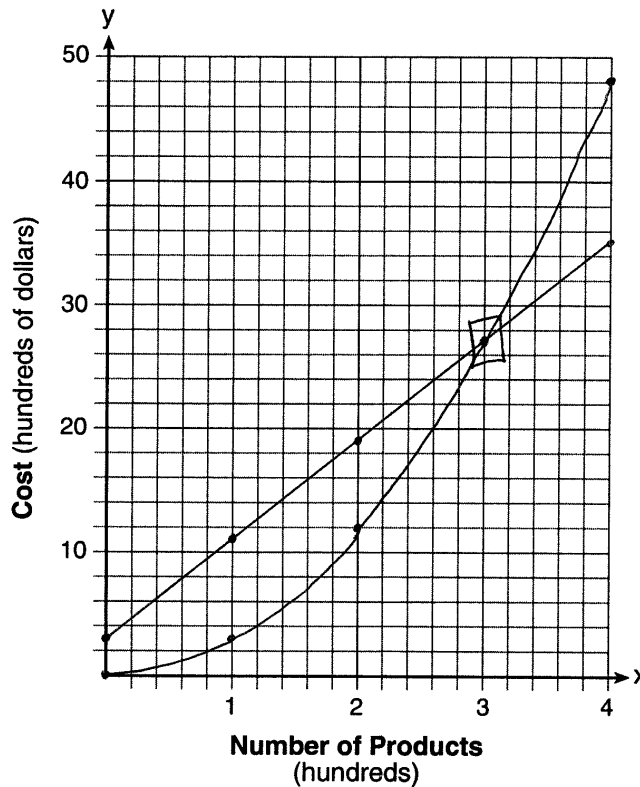
Question 37

37 A company is considering building a manufacturing plant. They determine the weekly production cost at site A to be $A(x) = 3x^2$ while the production cost at site B is $B(x) = 8x + 3$, where x represents the number of products, *in hundreds*, and $A(x)$ and $B(x)$ are the production costs, *in hundreds of dollars*.

Graph the production cost functions on the set of axes below and label them site A and site B.

$$y = 3x^2$$

x	y
0	0
1	3
2	12
3	27
4	48



$$y = 8x + 3$$

x	y
0	3
1	11
2	19
3	27
4	35

Question 37 is continued on the next page.

Question 37

Question 37 continued

State the positive value(s) of x for which the production costs at the two sites are equal.
Explain how you determined your answer.

$$3x^2 = 8x + 3$$

$$3x^2 - 8x - 3 = 0$$

$$(3x-3)(x+1) = 0$$

$$x = 1 \quad x = -1$$

If the company plans on manufacturing 200 products per week, which site should they use?
Justify your answer.

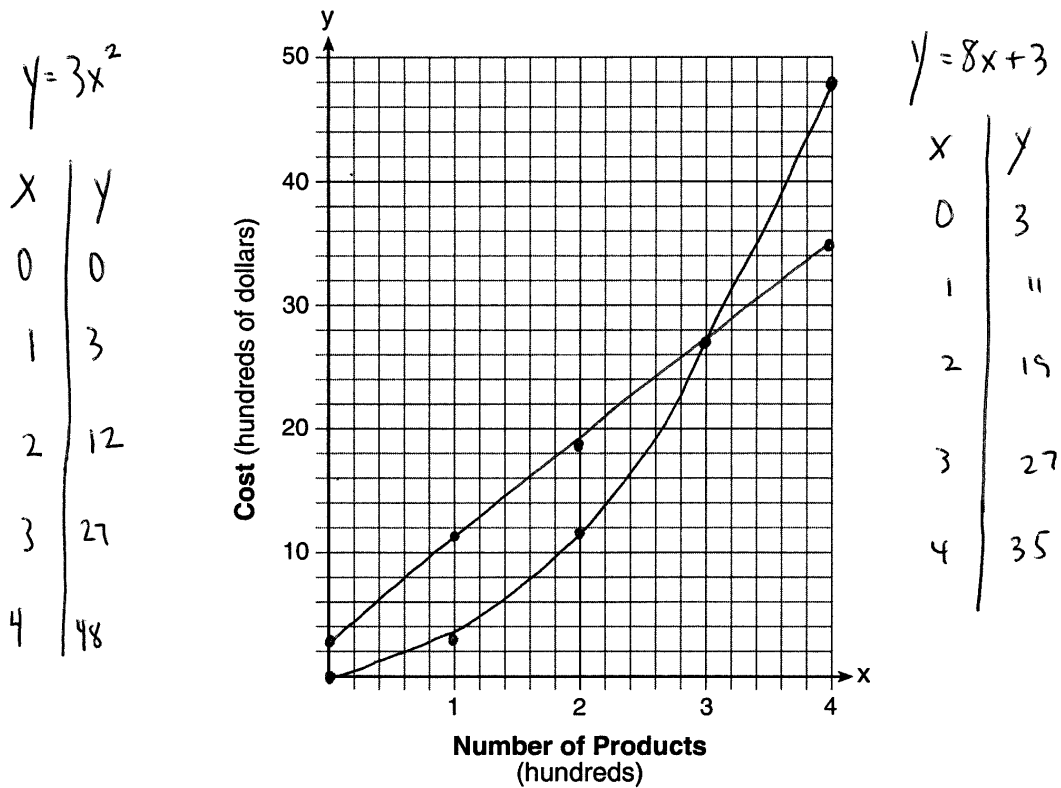
They should use site A. It will be cheaper to make products.

Score 3: The student graphed both functions correctly but did not label the graphs. The student set the equations equal, but made a factoring error and did not include an explanation. The student stated site A and gave a correct justification.

Question 37

37 A company is considering building a manufacturing plant. They determine the weekly production cost at site A to be $A(x) = 3x^2$ while the production cost at site B is $B(x) = 8x + 3$, where x represents the number of products, *in hundreds*, and $A(x)$ and $B(x)$ are the production costs, *in hundreds of dollars*.

Graph the production cost functions on the set of axes below and label them site A and site B.



Question 37 is continued on the next page.

Question 37

Question 37 continued

State the positive value(s) of x for which the production costs at the two sites are equal.
Explain how you determined your answer.

300

If the company plans on manufacturing 200 products per week, which site should they use?
Justify your answer.

$$\begin{aligned} A(x) &= 3x^2 \\ &= 3(200)^2 = 3(40000) \\ &= 120000 \end{aligned}$$

$$\begin{aligned} B(x) &= 8(200) + 3 \\ &= 1600 + 3 \\ &= 1603 \end{aligned}$$

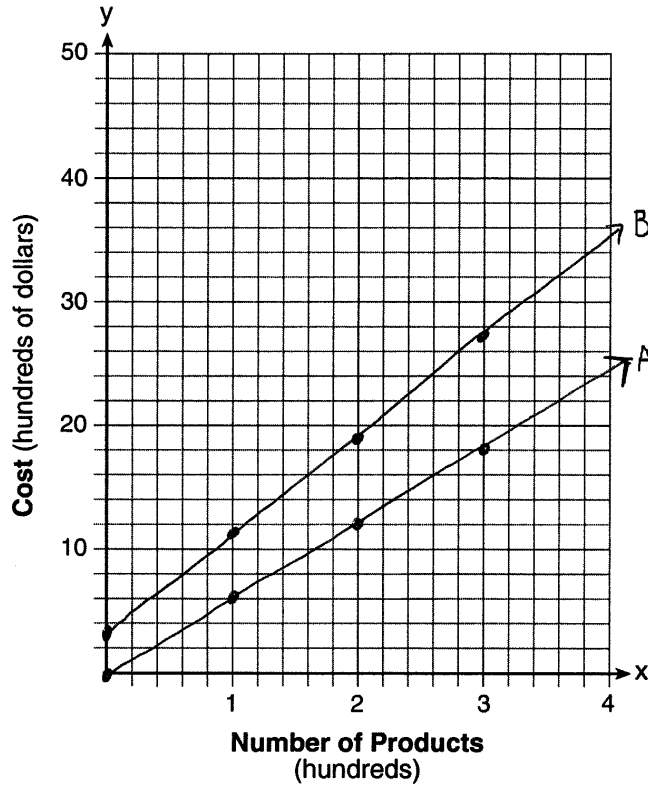
Site B is cheaper.

Score 2: The student graphed both functions correctly but did not label the graphs. The student stated 300 but did not provide an explanation. The student made an error by using 200 instead of 2 but stated site B based on the work shown.

Question 37

37 A company is considering building a manufacturing plant. They determine the weekly production cost at site A to be $A(x) = 3x^2$ while the production cost at site B is $B(x) = 8x + 3$, where x represents the number of products, *in hundreds*, and $A(x)$ and $B(x)$ are the production costs, *in hundreds of dollars*.

Graph the production cost functions on the set of axes below and label them site A and site B.



Question 37 is continued on the next page.

Question 37

Question 37 continued

State the positive value(s) of x for which the production costs at the two sites are equal.
Explain how you determined your answer.

The costs are equal because the slopes
are the same.

If the company plans on manufacturing 200 products per week, which site should they use?
Justify your answer.

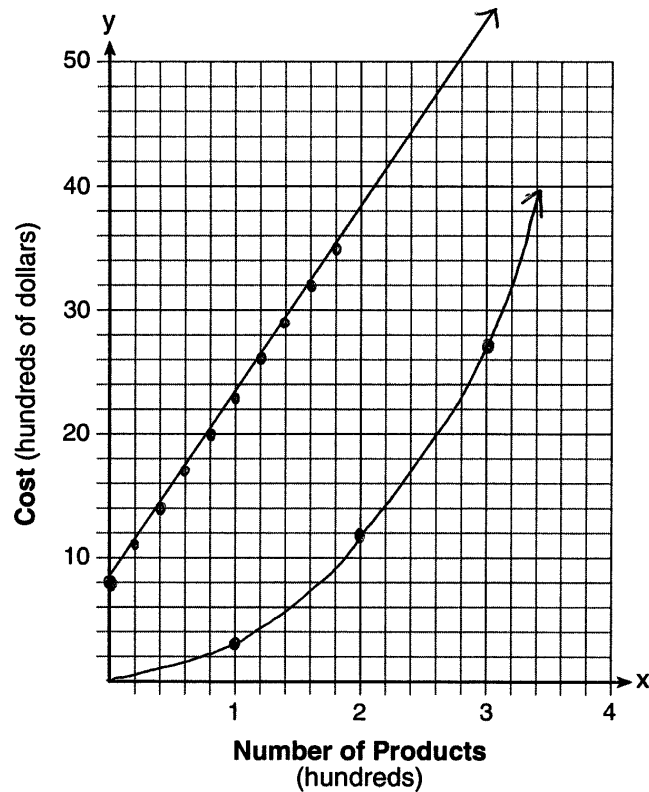
Site A has higher costs at 2 than B .

Score 1: The student graphed and labeled one function correctly. No further correct work was shown.

Question 37

37 A company is considering building a manufacturing plant. They determine the weekly production cost at site A to be $A(x) = 3x^2$ while the production cost at site B is $B(x) = 8x + 3$, where x represents the number of products, *in hundreds*, and $A(x)$ and $B(x)$ are the production costs, *in hundreds of dollars*.

Graph the production cost functions on the set of axes below and label them site A and site B.



Question 37 is continued on the next page.

Question 37

Question 37 continued

State the positive value(s) of x for which the production costs at the two sites are equal.
Explain how you determined your answer.

Graphs will cross at 6.

If the company plans on manufacturing 200 products per week, which site should they use?
Justify your answer.

site B

Score 0: The student graphed one function correctly, but did not label either graph. No further correct work was shown.